(The final) Quiz 6 (Wednesday, June 26th)

AndrewID: Name:

Total: 120 points. Full score: 100 points.

1. Let $\mathbb{X} = \{X_t\}_{t\geq 0}$ and $\mathbb{Y} = \{Y_t\}_{t\geq 0}$ be two independent Poisson processes with rates λ and μ respectively. Let $Z_t = X_t + Y_t$. Compute $E[X_t|Z_t = n]$.

2. Let $X \sim \text{Bernoulli}(|Y - 1/3|)$ where $Y = \mathbb{1}(Z \geq 0)$ and $Z \sim \mathcal{N}(0,1)$. Write down the pmf for X.

3. Let T_n be the n-th arrival of a Poisson process with rate λ. Compute $P(T_n \leq t)$.

4. Let $\mathbb{X}$ be a Markov chain with state space $S = \{0,1,2,3\}$. The transition probability matrix is

$$P = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.1 & 0.4 & 0.5 & 0 \\ 0 & 0 & 0.3 & 0.7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

List the communicating class(es).

5. Let $\mathbb{X}$ be a Markov chain with state space $S = \{0,1,2\}$. The transition probability matrix is

$$P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Does the MC admit a stationary distribution? If so, compute it.

6. Let $\mathbb{X}$ be a Markov chain with state space $S = \{0,1,2\}$. The transition probability matrix is

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Are the states periodic or aperiodic? What is $P^{(100)}$?