$$\min_{x \in C} \quad g(x) + h(x)$$

$$\min_{x} \quad g(x) + \underbrace{h(x) + I_C(x)}_{\tilde{h}(x)}$$

$$x^+ = \text{prox}_{\tilde{h}, t} \left(x - t\nabla g(x)\right)$$

$$= \operatorname*{argmin}_{z} \quad \frac{1}{2t} \|(x - t\nabla g(x)) - z\|_2^2 + \tilde{h}(z)$$

$$= \operatorname*{argmin}_{z \in C} \quad \frac{1}{2t} \|(\quad) - z\|_2^2 + h(z)$$

$$\min_{x} \quad f(x) \qquad \text{st} \quad \underline{Ax = b}$$

$$\min_{x,y} \quad x+y$$

$$x+y \geq 2 \leftarrow$$

$$x, y \geq 0$$

$$B = 2$$

$$\min_{x,y} \quad \underline{x+3y}$$

$$x+y \geq 2$$

$$x \geq 0$$

$$2y \geq 0$$

$$\underline{x+3y} \geq 2$$

$$B = 2$$

$$\min_{x,y} \quad px + qy$$

$$x+y \geq 2 \qquad a \geq 0$$

$$x \geq 0 \qquad b \geq 0$$

$$y \geq 0. \qquad c \geq 0$$

$$\underbrace{ax + ay + bx + cy} \geq 2a$$

$$(\underset{p}{a+b})x + (\underset{q}{a+c})y \geq 2a$$

lower bound $= 2a, \ \forall \ a, b, c \geq 0$ and $a+b = p$

$a+c = q$

$$\begin{array}{ll} \max & 2a \\ \text{s.t.} & a+b=p \\ & a+c=q \\ & a,b,c \geq 0 \end{array}$$

$$\min \quad px+qy$$

$$\begin{array}{ll} x \geq 0 & a \geq 0 \\ -y \geq -1 & b \geq 0 \\ 3x+y=2 & c \end{array}$$

$$ax - by + c(3x+y) \geq -b+2c$$

$$\underbrace{(a+3c)}_{p}x + \underbrace{(-b+c)}_{q}y \geq -b+2c$$

$$\begin{array}{ll} \max & -b+2c \\ & a+3c=p \\ & -b+c=q \\ & a,b \geq 0 \end{array}$$

$$\min \quad c^T x$$

$$\begin{array}{ll} Ax=b & u \\ Gx \leq h & v \end{array}$$

$v \geq 0$

$$u^T(Ax-b) + v^T(Gx-h) \leq 0$$

$$\underbrace{-(A^Tu+G^Tv)^T}_{c}x \geq -b^Tu - h^Tv$$

$$\min c^T x$$
$$Ax = b \qquad u$$
$$Gx \le h \qquad v \ge 0$$

$$\underline{c^T x} \geq \underline{c^T x + u^T(Ax-b) + v^T(Gx-h)}$$

for any feas x, $v \ge 0$. feas set C

$$\min_{x \in C} c^T x \geq \min_{x \in C} \left(c^T x + u^T(Ax-b) + v^T(Gx-h) \right)$$
$$\geq \min_{x} c^T x + u^T(Ax-b) + v^T(Gx-h)$$
$$\| $$
$$g(u,v)$$

$$\min_x c^T x + u^T(Ax-b) + v^T(Gx-h)$$

$$= \min_x (c + A^T u + G^T v)^T x - b^T u - h^T v$$

$$= \begin{cases} -b^T u - h^T v & \text{if } c = -A^T u - G^T v \\ -\infty & \text{else} \end{cases}$$

$$\max_{v \geq 0} \quad g(u,v)$$

$$= \max \quad -b^T u - h^T v$$

$$\text{s.t.} \quad c = -A^T u - G^T v$$
$$v \geq 0$$

$$\min \ f(x)$$

$h_i(x) \leq 0 \qquad i = 1 \ldots m \qquad u_i \geq 0.$

$l_j(x) = 0. \qquad j = 1 \ldots r \qquad v_i$

$$L(x,u,v) = f(x) + \sum u_i h_i(x) + \sum v_j l_j(x)$$

if x is feas., then $f(x) \geq L(x,u,v)$

$u_i \geq 0$

$$f^* = \min_{x \in C} f(x)$$

$$\geq \min_{x \in C} L(x,u,v) \qquad \text{provided } u \geq 0$$

$$\geq \min_{x} L(x,u,v)$$

$$= g(u,v)$$

$$\min \quad \tfrac{1}{2}x^T Q x + c^T x$$

$$\text{s.t.} \quad Ax = b \qquad v$$

$$-x \leq 0. \qquad u \geq 0$$

$$L(x,u,v) = \tfrac{1}{2}x^T Q x + c^T x - u^T x + v^T(Ax - b)$$

$$g(u,v) = \min_x \; L(x,u,v)$$

$$= \min_x \; \tfrac{1}{2}x^T Q x + (c - u + A^T v)^T x - b^T v$$

$ax^2 + bx + c$

$x = \frac{-b}{2a}$

$$x = -Q^{-1}(c - u + A^T v)$$

$$g(u,v) = \tfrac{1}{2}\left(-Q^{-1}(c - u + A^T v)\right)^T Q \left(-Q^{-1}(c - u + A^T v)\right) + -(c - u + A^T v)^T Q^{-1} \cdot (c - u + A^T v) - b^T v$$

$$= -\tfrac{1}{2}(c - u + A^T v)^T Q^{-1}(c - u + A^T v) - b^T v$$

$\forall\, v,\; u \geq 0$