$$f^* = \min_{x \in C} f(x)$$

$$\geq \min_{x \in C} L(x,u,v)$$

$$\geq \min_{x \in \mathbb{R}^n} L(x,u,v)$$

$$:= g(u,v) \quad \text{for any } u \geq 0, \ v$$

---

primal	dual
$\min_{x \in C} f(x)$	$\max_{u \geq 0, v} g(u,v)$

$$f^* \geq g^*$$

weak duality

always holds, even true if nonconvex

$g(u,v)$ is always concave

$$\max g(u,v) \iff \min \underline{-g(u,v)}$$

$$g(u,v) = \min_x L(x,u,v) = -\max_x \left\{ \underbrace{-f(x) - \sum u_i h_i(x) - \sum v_j l_j(x)}_{\text{convex}} \right\}$$

$$L(x,u) = \underbrace{x^4 - 50x^2 + 160x - ux}_{} - 4.5u$$

$$g(u) = \min_x \qquad \parallel$$

$$=$$

$$f^* = g^*$$

$$\begin{aligned} \min \quad & f(x) && \leftarrow \text{convex} \\ & h_i(x) \leq 0 && \leftarrow \text{convex} \\ & l_j(x) = 0 && \leftarrow \text{affine} \end{aligned}$$

$\exists x$ s.t. $h_i(x) < 0$ all i
and $l_j(x) = 0$ all j

$h_i(x) < 0$ all i such that h_i nonaffine
$l_j(x) = 0$ all j

LPs: strong duality if problem is feas

$\left(\begin{array}{ll} \exists x \; h_i(x) \leq 0 & \text{all } i \\ \text{s.t. } l_j(x) = 0 & \text{all } j \end{array}\right)$

if LP is feas, then strong duality $f^* = g^*$
if DLP is feas, then strong duality $g^* = f^*$