$$D = \begin{bmatrix} - D_1 - \\ - D_2 - \\ \vdots \\ - D_m - \end{bmatrix}$$

$D\beta$ sparse $\iff$ $D_j \beta = 0$ many j

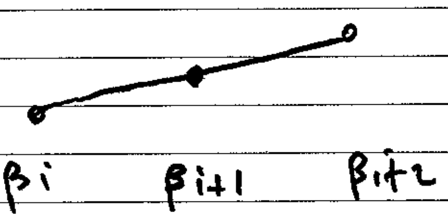
$$D = \begin{bmatrix} - e_1 - \\ - e_2 - \\ \vdots \\ - e_n - \end{bmatrix}$$

$$\sum (y_i - \beta_i)^2 + \lambda \sum |\beta_i - \beta_{i+1}|$$

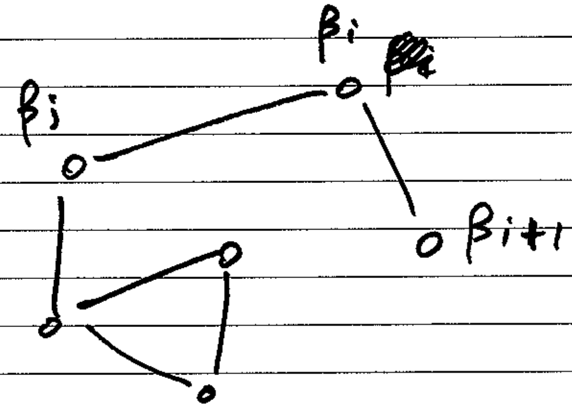
$$p_i = \frac{e^{\beta_i}}{1 + e^{\beta_i}}$$

$$\beta_i - 2\beta_{i+1} + \beta_{i+2} = 0$$

$$\iff \quad \beta_{i+1} = \frac{\beta_i + \beta_{i+2}}{2}$$



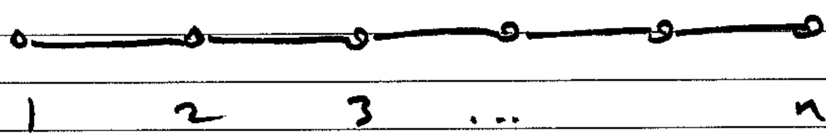
$$D \quad \underset{\substack{\uparrow \\ \#\text{ of edges}}}{m} \times \underset{\substack{\uparrow \\ \#\text{ of nodes}}}{n}$$



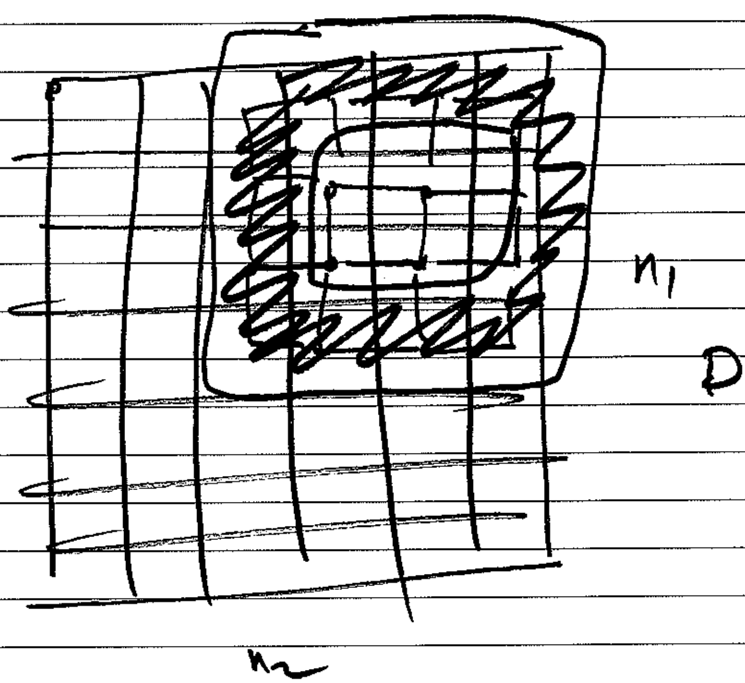
$$D = \begin{bmatrix} - & D_1 & - \\ & \vdots & \\ - & D_m & - \end{bmatrix}$$

$e_l = (i, j)$

$$\begin{array}{cccccccccc} D_l = (& 0 & \dots & -1 & \dots & 1 & \dots & 0 &) \\ & & & i & & j & & & \end{array}$$



$$y_i = \frac{\#\text{ of cases of flu in state } i}{\#\text{ of ppl in state } i}$$



$n = n_1 \cdot n_2$

$\beta = D\theta \iff A\beta = 0$

for some $\theta \in \mathbb{R}^k$

$\iff f(D\theta) + \lambda \|D\theta\|_1$

$\iff g(\theta) + \lambda \|D\theta\|_1$

$\hat{\beta} = D\hat{\theta}$.

$\nabla f(\beta) + \lambda D^T \gamma$

$\gamma \in \|x\|_1 \Big|_{x = D\beta}$

Chain rule:

$g(x) = f(Ax+b)$

$\partial g(x) = A^T \partial f(Ax+b)$

$\beta \Leftarrow \beta + \nabla f(\beta) + \lambda D^T \gamma$

$\min_{\beta} \; f(\beta) + \lambda \|D\beta\|_1$

$\min_{z,\beta} \; f(\beta) + \lambda \|z\|_1 \quad \text{s.t.} \quad z = D\beta$

$$\boxed{(DD^T)^{-1} b}$$

$$\nabla f(\hat{\beta}) + D^T \hat{u} = 0.$$

$$\hat{u}_i \in \begin{cases} \{\lambda\} & \text{when } (D\hat{\beta})_i > 0 \\ \{-\lambda\} & \text{when } (D\hat{\beta})_i < 0 \\ [-\lambda, \lambda] & \text{else} \end{cases}$$

$$-\lambda < \hat{u}_i < \lambda \implies (D\hat{\beta})_i = 0.$$

$$\|u\|_\infty \le \lambda \iff -\lambda \le u_i \le \lambda \quad i = 1 \ldots m$$

$$\min \ f^*(-D^T u) + \frac{1}{t}\phi(u)$$

$$\phi(u) = -\sum \left[\log(\lambda + u_i) + \log(\lambda - u_i) \right]$$

$$F(u) = f^*(-D^T u) + \frac{1}{t}\phi(u)$$

$$\nabla^2 F(u) = D \nabla^2 f^* D^T + \frac{1}{t}\nabla^2 \phi$$

$$\nabla F(u) = D \nabla f^* + \frac{1}{t}\nabla \phi$$