suppose f convex, diff.

$$f(x + \delta \cdot e_i) \geq f(x) \quad \text{for all } \delta, i$$

$$\overset{?}{\Longrightarrow} \quad x \text{ minimizer of } f$$

<u>YES</u>

$$\nabla_i f(x) = 0 \quad \text{for all } i$$

$$\nabla f(x) = (\nabla_1 f(x), \ldots, \nabla_n f(x)) = 0$$

$$\Rightarrow x \text{ minimizer of } f$$

suppose f convex ...

<u>NO</u>

suppose $$f(x) = \underbrace{g(x)}_{\substack{\text{smooth} \\ \text{convex}}} + \underbrace{\sum_{i=1}^{n} h_i(x_i)}_{\substack{\text{nonsmooth} \\ \text{convex}}}$$

x_i minimizes $g(y) + \sum_{j=1}^{n} h_j(y_j)$ along y_i

then $$\nabla_i g(x) + \partial h_i(x_i) \ni 0.$$

ie. $$-\nabla_i g(x) \in \partial h_i(x_i)$$

$$h_i(y_i) \geq h_i(x_i) - \nabla_i g(x) \cdot (y_i - x_i)$$

for any y_i

$$\nabla_i g(x) \cdot (y_i - x_i) + h_i(y_i) - h_i(x_i) \geq 0.$$

for any y_i

$$f(y) - f(x) = g(y) - g(x) + \sum_{i=1}^{n} h_i(y_i) - h_i(x_i)$$

$$\geq \nabla g(x)^T (y-x) + \sum_{i=1}^{n} h_i(y_i) - h_i(x_i)$$

$$= \sum_{i=1}^{n} \Big[\underbrace{\nabla_i g(x)(y_i - x_i) + h_i(y_i) - h_i(x_i)}_{\geq 0} \Big]$$

$$\geq 0$$

$$0 = \nabla_i f(\beta)$$

$$= X_i^T (X\beta - y)$$

$$= X_i^T (X_i \beta_i + X_{-i}\beta_{-i} - y)$$

$$\beta_i = \frac{X_i^T (y - X_{-i}\beta_{-i})}{X_i^T X_i}$$

$$\beta^{+} \Leftarrow \beta + t \cdot X^T (y - X\beta)$$

$$\beta_i \Leftarrow \frac{X_i^T (y - X_{-i}\beta_{-i})}{X_i^T X_i}$$

$$= \frac{X_i^T (\overbrace{y - X\beta} + X_i \beta_i)}{X_i^T X_i}$$

$$= \frac{X_i^T r}{X_i^T X_i} + \beta_i$$

$$\|\beta\|_1 = \sum_{i=1}^{p} |\beta_i| \quad \checkmark \quad \text{separable}$$

$$\|\beta\|_2^2 = \sum_{i=1}^{p} \beta_i^2 \quad \checkmark$$

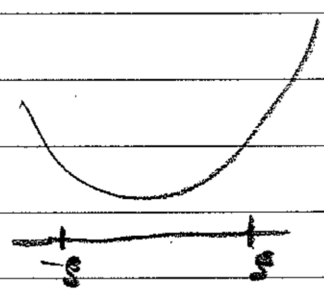
$$\|\beta\|_2 = \sqrt{\sum_{i=1}^{p} \beta_i^2}$$

$$\min_{\beta} \ \frac{1}{2}\|y - X\beta\|_2^2 \quad \text{s.t.} \quad \|\beta\|_\infty \le s$$

$$\Longleftrightarrow \min_{\beta} \ \frac{1}{2}\|y - X\beta\|_2^2 + \sum_{i=1}^{p} \underbrace{\mathbb{1}\{|\beta_i| \le s\}}_{h_i(\beta_i)}$$

$$\min_{\beta_i} \ \frac{1}{2}\|y - X\beta\|_2^2 \quad \text{s.t.} \quad |\beta_i| \le s$$

$$\Longleftrightarrow \min_{\beta_i} \ a\beta_i^2 + b\beta_i + c \quad \text{s.t.} \quad -s \le \beta_i \le s$$



$$\min_{\beta} \ \frac{1}{2}\|y - X\beta\|_2^2 + \lambda\|\beta\|_\infty \quad \times$$