

| | Gradient | Subgrad | Prox grad | Newton | Conj grad | quasi-Newton |
|------------------------|---|--------------------------------|---|---|---|--|
| Criterion | smooth f | any f | smooth + simple, $f = g + h$ | doubly smooth f | doubly smooth f | doubly smooth f |
| Constraints | projection onto constraint set | projection onto constraint set | constrained prox operator | equality constraints | unconstrained | unconstrained |
| Opti parameters | fixed step size ($t \leq 1/L$) or line search | diminishing step sizes | fixed step size ($t \leq 1/L$) or line search | pure step size ($t = 1$) or line search | FR & PR: line search, versions that use the Hessian: fixed step size | DFP & BFGS: line search |
| Iteration cost | cheap (compute gradient) | cheap (compute subgradient) | moderately cheap (evaluate prox) | moderate to expensive (compute Hessian and solve linear system) | moderately cheap (compute gradients, inner products) | moderately cheap (compute gradients, inner products; no matrix inversion, but storage for estimated inverse Hessian) |
| Rate | $O(\frac{1}{\epsilon})$ [$O(\frac{1}{\sqrt{\epsilon}})$ with acceleration, $O(\log(\frac{1}{\epsilon}))$ with strong convexity] | $O(\frac{1}{\epsilon^2})$ | $O(\frac{1}{\epsilon})$ [$O(\frac{1}{\sqrt{\epsilon}})$ with acceleration, $O(\log(\frac{1}{\epsilon}))$ with strong convexity] | $O(\log \log(\frac{1}{\epsilon}))$ (quadratic rate) | superlinear rate, n -step quadratic rate (n steps are as effective as one Newton step) | superlinear rate, n -step quadratic rate (n steps are as effective as one Newton step) |

| | Barrier method | Primal-dual IPM | ADMM | Coord desc |
|------------------------|---|--|---|---|
| Criterion | doubly smooth f | doubly smooth f | block separable, $f(x, z) = g(x) + h(z)$ | smooth + component-wise separable |
| Constraints | doubly smooth h_i (ineq constraints) | doubly smooth h_i (ineq constraints) | equality constraints (always) & ineq constraints (sometimes) | component-wise separable constraints |
| Opti parameters | inner loop: fixed step size or use line search, outer loop: diverging barrier parameter | line search for step size & diverging barrier parameter | fixed augmented Lagrange parameter (theory), or varied by iteration (practice) | none! |
| Iteration cost | expensive to very expensive (one iteration solves one smoothed problem, by Newton) | moderate to expensive (one iteration performs one Newton step) | cheap to expensive (one iteration solves two subproblems, makes a dual step) | cheap to expensive (one iteration performs a full cycle of component minimizations) |
| Rate | $O(\log(\frac{1}{\epsilon}))$ (both in terms of iterations and Newton steps) | $O(\log(\frac{1}{\epsilon}))$ | not known in general, but known in special cases; practically tends to behave like a first-order method | not known in general, but known in special cases; practically tends to behave faster than first-order methods |