$$\begin{array}{llll} \min & px + qy & & \\ \text{s.t.} & x + y \geq 2 & a & \\ & x \geq 0 & b & a,b,c \geq 0 \\ & y \geq 0 & c & \end{array}$$

$$\left.\begin{array}{r} ax + ay \geq 2a \\ bx \geq 0 \\ cy \geq 0 \end{array}\right\}$$

$$\underbrace{(a+b)}_{=\,p} x + \underbrace{(a+c)}_{=\,q} y \geq 2a$$

for any $a, b, c \geq 0$, where
$p = a+b$
$q = a+c$
have a lower bd of $B = 2a$ — to get tightest lower bd, maximize this

$x + y \geq 2 \qquad 3x + 6y \geq 10$

$x + y + 3x + 6y \geq 2 + 10 = 12$

$$\begin{array}{lll} \min & px + qy & \\ \text{s.t.} & x \geq 0 & a \geq 0 \\ & y \leq 1 \iff -y \geq -1 & b \geq 0 \\ & 3x + y = 2 & c \end{array}$$

$ax - by + c(3x+y) \geq -b + 2c$

$$\underbrace{(a+3c)}_{=\,p} + \underbrace{(-b+c)}_{=\,q} y \geq -b + 2c$$

$$\sum u_i \underbrace{(a_i^T x - b_i)}_{=\,0} + \sum \underset{\geq 0}{v_j} \underbrace{(g_j^T x - h_j)}_{\leq 0} \leq 0$$

Val max flow prob $\leq$ val of its dual LP

$\leq$ val of min cut prob.

$=$

by max flow min cut thm, so all three are actually equalities

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set $v \geq 0$, $c^T x \geq L(x,u,v)$ all feasible x

C feasible set

$$f^* \geq \min_{x \in C} L(x,u,v)$$

$$\geq \min_x L(x,u,v)$$

$$= g(u,v)$$

$$\min_x L(x,u,v) = \min_x c^T x + u^T(Ax-b) + v^T(Gx-h)$$

$$= \min_x (c + A^T u + G^T v)^T x - b^T u - h^T v$$

$$= \begin{cases} -\infty & \text{if } c + A^T u + G^T v \neq 0 \\ -b^T u - h^T v & \text{else} \end{cases}$$

x is fixed

$$\max \; y^T (P^T x) \quad \text{over } y \geq 0, \; 1^T y = 1$$

$$= \max_{i=1..n} (P^T x)_i$$

R's payoffs f_1^* scenario 1

f_2^* scenario 2

conclude $f_1^* \geq f_2^*$

actually $f_1^* = f_2^*$!

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min t

$x \geq 0 \iff -x \leq 0$ $\quad u \geq 0$

$1^T x = 1$ $\quad v$

$P^T x \leq t$ $\quad y \geq 0$

$$L(x,t,u,v,y) = t - u^T x + v^T(1 - 1^T x) + y^T(P^T x - t)$$

$$= (-u - v1 + Py)^T x + (1 - y^T 1)t + v$$

$$\begin{aligned} \max_{v,y,u} \quad & v \\ \text{s.t.} \quad & y^T 1 = 1 \\ & Py = u + v1 \\ & u \geq 0 \\ & y \geq 0 \end{aligned}$$

eliminate slack vars