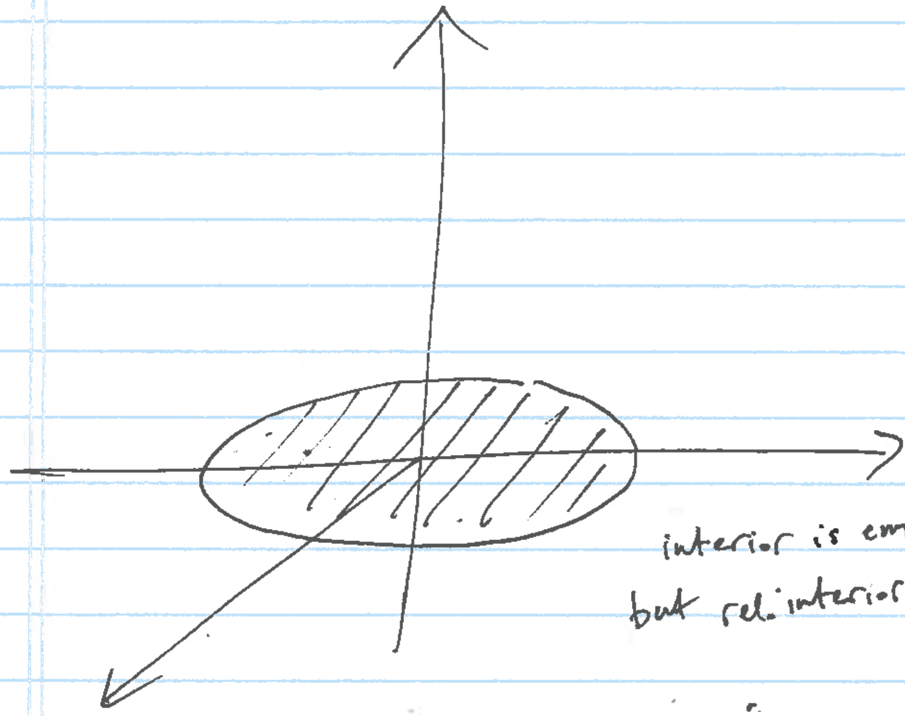

interior is empty
but rel. interior is not

Necessity of KKT

$$f^* = g^*$$

$$= \min_{x} \; f(x) + \sum u_i^* h_i(x) + \sum v_j^* l_j(x)$$

$$\le f(x^*) + \sum \underbrace{u_i^*}_{\ge 0} \underbrace{h_i(x^*)}_{\le 0} + \sum v_j^* \underbrace{\cancel{l_j(x^*)}}_{0}$$

$$\le f(x^*)$$

Hence

actually an equality $\Rightarrow$ stat. condition

actually an equality $\Rightarrow$ comp. slackness

Sufficiency of KKT

$$g^* = g(u^*, v^*) = \min_x L(x, u^*, v^*)$$

$$\overset{\text{stat. cond}}{=} L(x^*, u^*, v^*)$$

$$= f(x^*) + \underbrace{\sum u_i^* h_i(x^*)}_{0} + \underbrace{\cancel{\sum v_j^* l_j(x^*)}}_{0}$$

$$\overset{\text{comp. slack}}{=} f(x^*)$$

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subg. of F at x

$$0 \in \partial F(x)$$

$$\iff F(y) \geq F(x) + 0^T(y - x) \quad \text{all } y$$

$$\iff F(y) \geq F(x) \quad \text{all } y$$

$$\min_x f(x) + \mathbb{1}\{x \in C\}$$

$Ax = 0 \iff x \in Ny$

rewrite problem in terms of y

$$\min_y \ \frac{1}{2} y^T N^T Q N y + c^T N y$$

at sol. y^*, take $x^* = Ny^*$

QP with equality constraints

stat cond: $\nabla_x \left(\frac{1}{2} x^T Q x + c^T x + u^T A x \right) = 0.$

$$Qx + A^T u = -c$$

comp. slack: $\emptyset$

prim feas: $Ax = 0$

dual feas: $\emptyset$

$$\begin{bmatrix} Q & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} = \begin{bmatrix} -c \\ 0 \end{bmatrix}$$

SVM

$$\nabla_\beta \left(\frac{1}{2}\|\beta\|_2^2 + \sum w_i (1 - \xi_i - y_i x_i^T \beta) \right) = 0$$

stat.

$$\beta - \sum w_i y_i x_i = 0$$

$$\boxed{\beta = \sum w_i y_i x_i}$$

$$\nabla_{\beta_0} \left(\sum w_i (1 - \xi_i - y_i \beta_0) \right) = 0$$

$$\boxed{\sum w_i y_i = 0}$$

$$\nabla_\xi \left(C \sum \xi_i - \sum v_i \xi_i + \sum w_i (1 - \xi_i - y_i (x_i^T \beta + \beta_0)) \right) = 0$$

$$\nabla_\xi \left(C \mathbb{1}^T \xi - v^T \xi + w^T (\mathbb{1} - \xi) \right) = 0$$

$$C\mathbb{1} - v - w = 0$$

$$\boxed{w = C\mathbb{1} - v.} \implies 0 \le w \le C$$

comp slack: $\quad v_i \xi_i = 0 \quad$ all i

$\quad w_i(\underbrace{1 - \xi_i - y_i(x_i^T\beta + \beta_0)}) = 0 \quad$ all i $\longrightarrow$ $\left(\begin{array}{l}\text{if } 1-\xi_i > y_i(x_i^T\beta+\beta_0) \\ \text{then } w_i = 0\end{array}\right)$

primal feas: $\quad$ as in SVM

dual feas: $\quad v \geq 0, \ w \geq 0$

Hessian $\quad X^T \nabla^2 f(X\beta) X$

$X\gamma = 0 \quad , \gamma \neq 0$

Ex: $\quad f(u) = \|y-u\|_2^2$

loss is $\quad f(X\beta) = \|y - X\beta\|_2^2$

$j \notin S \implies X_j^T \nabla f(X\beta) \in (-\lambda, \lambda)$

$\implies$ ~~[illegible]~~ $s_j \in (-1, 1)$

$\implies \beta_j = 0$

assume X_S is rank def.

$-X_j^T \nabla f(X\beta) = \lambda s_j \qquad j = 1 \ldots n$

$s_i X_i$ is an affine combo of $s_j X_j$
$j \in S \setminus \{i\}$