Mahalonobis norm in $A \succeq 0$

$$\|x\|_A = (x^T A x)^{1/2}$$

Newton decrement:

$$\lambda(x) = \|\nabla f(x)\|_{[\nabla^2 f(x)]^{-1}}$$

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if $A \succ 0$, and $\|x\|_A = 0$

then $x = 0$.

$\nabla^2 f$ Lipschitz means

$$\|\nabla^2 f(x) - \nabla^2 f(y)\|_F \leq M \|x-y\|_2$$

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compare $\log(1/\varepsilon)$ for grad desc

and $\log\log(1/\varepsilon)$ for Newton method

Huge difference! If $\varepsilon = 2^{-32}$

$$\log\log(1/\varepsilon) = \log\log(2^{2^5}) = 5.$$

$$f(x) = -\log(x)$$

$$f'(x) = -\frac{1}{x}$$

$$f''(x) = \frac{1}{x^2}$$

$$f'''(x) = -\frac{2}{x^3}$$

$$|f'''(x)| = 2 f''(x)^{3/2}$$

Equality const. Newton

start at x st $Ax = b$

update in dir v st. $Av = 0$

therefore $Ax^+ = Ax + Av = b$

v is given by usual quad approx.

st. $Av = 0$