$$\min_{x \in C} \; g(x) + h(x)$$

$$\min_{x} \; \underbrace{g(x)} + \underbrace{h(x) + I_C(x)}$$

$\uparrow$

need prox of this

$$\text{prox}_t(x) = \underset{z}{\text{argmin}} \; \frac{1}{2t} \|x - z\|_2^2 + h(z) + I_C(z)$$

$$= \underset{z \in C}{\text{argmin}} \; \frac{1}{2t} \|x - z\|_2^2 + h(z)$$

e.g. $C = \{x : Ax \le b\}$

$h(x) = \|x\|_1$

prox of h is easy here; but not constrained prox

$\min f(x)$

s.t. $\|x\|_1 \le R$

$$x^+ = P_{\{z : \|z\|_1 \le R\}}(x - t\nabla g(x))$$

$$\min_x \; f(x) + \|x\|_1 \iff \min_{x,z} \; f(x) + 1^T z$$

$$-z \le x \le z$$

$\min f(x)$

s.t. $h_i(x) \le 0$

approximate with $\displaystyle \min_x \; f(x) - \frac{1}{t} \sum \log(-h_i(x))$

check if

$f(x+tv) > f(x) + \alpha t \nabla f(x)^T v$

passes, shrink t

else, stop.

E.g. compare for this problem

$$\min_{\beta} \sum_{i=1}^{n} -y_i x_i^T \beta + \log(1+e^{x_i^T\beta}) + \lambda \|\beta\|_1$$

all the methods that are applicable.

strengths & weaknesses

(f) $x_1 + x_2 = -2x_1 + x_3 = tb$

$\log(\_ + \_) = \log(2e^{tb})$

$=$

$\log(\Sigma e^{x_i})$ log-sum-exp

Taylor series with <u>Remainder</u>

$$f(y) = f(x) + f'(x)(y-x) + \frac{f''(x)}{2}(y-x)^2$$
$$+ \quad \text{Rem}$$

---

eg.

$$f(y) = f(x) + f'(x)(y-x) + \frac{f''(z)}{2}(y-x)^2$$

for some z between x and y.

$$\min_X \|X - R\|_F^2 \quad \text{s.t. } \operatorname{rank}(R) = k$$

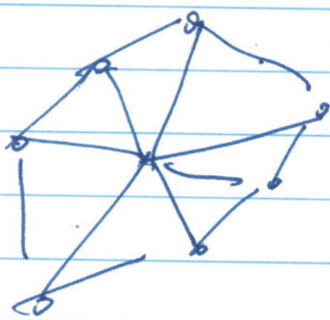
$$\Longleftrightarrow \min_Z \|X - XZ\|_F^2 \quad \text{s.t. } \operatorname{rank}(Z) = k$$
Z is a proj.

$$\|X - R\|_F^2 = \|X - R_1\|_F^2 + \|R_2\|_F^2$$

$R = R_1 + R_2$

$\uparrow$

aligned with col space of X



$$\max_{Z} \ \langle X, Z \rangle$$

s.t. $\operatorname{rank}(Z) = k$

Z proj.

$$\iff \max_{Z} \ \langle X, Z \rangle$$

s.t. $\operatorname{tr}(Z) = k$

$0 \preceq Z \preceq I$

Prox operator. How to derive?

1. Does it decompose? Across coordinates? Block?
2. Argue based on first principles?
3. Take subgradients, set equal to 0.

$$\min_{z} \ \tfrac{1}{2}\|x-z\|_2^2 + t\,h(z)$$

$$z - x + t\,\partial h(z) \ni 0$$

e.g. l_1 norm

$$\min_{z} \ \tfrac{1}{2}\|x-z\|_2^2 + t\|z\|_1$$

$$\iff \min_{z_i} \ \tfrac{1}{2}(x_i - z_i)^2 + t|z_i|$$

$i = 1, \ldots, n$

e.g. l_2 norm

$$\min_{z} \ \tfrac{1}{2}\|x-z\|_2^2 + t\|z\|_2$$

$z = cx$