



$$\min_{\beta} f(\beta) + \lambda \|D\beta\|_1 \quad \left. \vphantom{\min_{\beta}} \right\} \beta^+ = \beta - t g$$

$$\nabla f(\beta) + \lambda \underbrace{D^T \gamma}_{\in \partial \| \cdot \|_1}, \text{ where } \gamma \in \partial \| \cdot \|_1 \Big|_{D\beta}$$

$$\nabla f(\beta) + \lambda \sum D_j \gamma_j$$

$$\begin{aligned} \gamma_j &= -1 && \text{if } (D\beta)_j < 0 \\ &= 1 && (D\beta)_j > 0 \\ &\in [-1, 1] && (D\beta)_j = 0 \end{aligned}$$

valid choice of sub-grad is

$$\nabla f(\beta) + \lambda \sum_{j \in S} D_j \gamma_j, \quad S = \{j : (D\beta)_j \neq 0\}$$



$$\|x\|_p = \max_{\|y\|_q \leq 1} y^T x \quad \frac{1}{p} + \frac{1}{q} = 1.$$

$$\partial \|x\|_p = \operatorname{argmax}_{\|y\|_q \leq 1} y^T x$$

$$\partial \|x\|_2 = \frac{x}{\|x\|_2} \quad x \neq 0.$$

$$\in \{y : \|y\|_2 \leq 1\} \quad x = 0.$$

$$y_{11} \approx (\beta_1 y_{10} + \beta_2 y_9) + (\beta_3 y_8 + \beta_4 y_7) + \dots (\beta_9 y_2 + \beta_{10} y_1)$$

$$= (\beta_1 x_{11,1} + \beta_2 x_{11,2}) + \dots (\beta_9 x_{11,9} + \beta_{10} x_{11,10})$$

X matrix in row 11