Zoom out

Convex

All optimization

different initializations lead to very different solutions
\[
\begin{align*}
\min \quad & c^T x + f(y) \\
\text{s.t.} \quad & Ax = b \\
\quad & x \geq 0 \quad \text{nonconvex} \\
\quad & f(y) = 0
\end{align*}
\]

\[\|x\|_2 = 1 \text{ as constraint} \Rightarrow \text{nonconvex problem}\]

\[\Rightarrow \text{convex problem}\]

Not generally the same, but are the same if ineq. is tight at solution of latter problem
\[ \min_{Z} \| S - ZS \|_F^2 \quad \text{Z is a projection} \]
\[ \text{rank}(Z) = k \]
\[ \text{rank} \left( \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right) = 2 \]

\[ \min \sum (y_i - f(x_i))^2 \]
\[ + \int_a^b (f''(x))^2 \, dx \]
\[ k = 3 \]

Cubic smoothing splines

\[ \sum x_i \in \mathbb{R}^p : \| x_i \|_1 = 1, \quad i = 1, \ldots, p \]