1

$$B = U \Sigma V^T$$

$$= \begin{bmatrix} u_1 \cdots u_r \end{bmatrix} \begin{bmatrix} \sigma_{11} & & \\ & \ddots & \\ & & \sigma_{rr} \end{bmatrix} \begin{bmatrix} v_1^T \\ \vdots \\ v_r^T \end{bmatrix}$$

$$S_\lambda(B) = U \Sigma_\lambda V^T$$

$$= \begin{bmatrix} u_1 \cdots u_r \end{bmatrix} \begin{bmatrix} \sigma_{11}-\lambda & & & & \\ & \ddots & & & \\ & & \sigma_{kk}-\lambda & & \\ & & & 0 & \\ & & & & \ddots \\ & & & & & 0 \end{bmatrix} \begin{bmatrix} - v_1^T - \\ \vdots \\ - v_r^T - \end{bmatrix}$$

$$= \begin{bmatrix} u_1 \cdots u_k \end{bmatrix} \begin{bmatrix} \sigma_{11}-\lambda & & \\ & \ddots & \\ & & \sigma_{kk}-\lambda \end{bmatrix} \begin{bmatrix} v_1^T \\ \vdots \\ v_k^T \end{bmatrix}$$

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criterion of prox: $\frac{1}{2} \| B - Z \|_F^2 + \lambda t \cdot \| Z \|_*$

$$Z - B + \lambda t \cdot \Gamma = 0$$

for some $\Gamma \in \partial \| Z \|_*$

check this when $Z = S_{\lambda t}(B) = U \Sigma_{\lambda t} V^T$

where $B = U \Sigma V^T$

to get subgradients of $\| Z \|_*$

use $\| Z \|_* = \max_{\| Y \|_{op} \leq 1} Z \bullet Y$

$$= \max_{\| Y \|_{op} \leq 1} tr(Z^T Y)$$

②

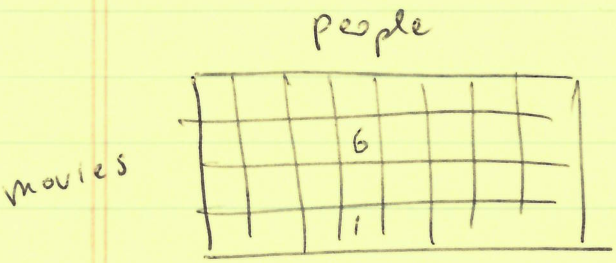
while

$$g\big(x - t\,G_t(x)\big) > g(x) - t\,\nabla g(x)^T G_t(x) + \frac{t}{2}\|G_t(x)\|_2^2$$

update $t = \beta t$

BUT in order to evaluate $G_t(x)$

we need $\mathrm{prox}_t(\cdot)$!



Y_{ij}

$$\underset{m \times n}{B} = \underset{m \times k}{U}\ \underset{k \times n}{V^T}$$

$\mathrm{rank}(B) = \sum 1\{\sigma_i(B) \neq 0\}$	$\|b\|_0 = \sum 1\{b_i \neq 0\}$
$\|B\|_* = \sum \sigma_i(B)$	$\|b\|_1 = \sum \lvert b_i \rvert$
matrix world	vector world

③

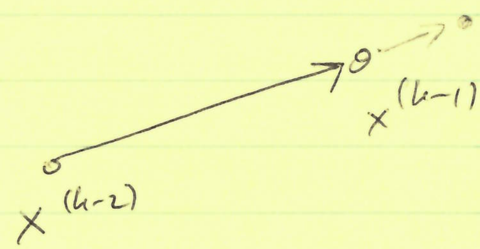
$$t=1$$

$$B^+ = S_\lambda\left(B + P_\Omega(Y) - P_\Omega(B)\right)$$

$$= S_\lambda\left(P_\Omega(Y) + P_{\Omega^c}(B)\right)$$

$$\begin{bmatrix} B_{ij} & \\ Y_{ij} & B_{ij} \\ B_{ij} & B_{ij} \\ B_{ij} & Y_{ij} \end{bmatrix}$$

soft-impute



$O(1/\sqrt{\varepsilon})$, $O(1/\varepsilon)$, $O(1/\varepsilon^2)$