

$x_0$  satisfies  $Ax_0 = b$   
columns of  $M$  span  $\text{null}(A)$

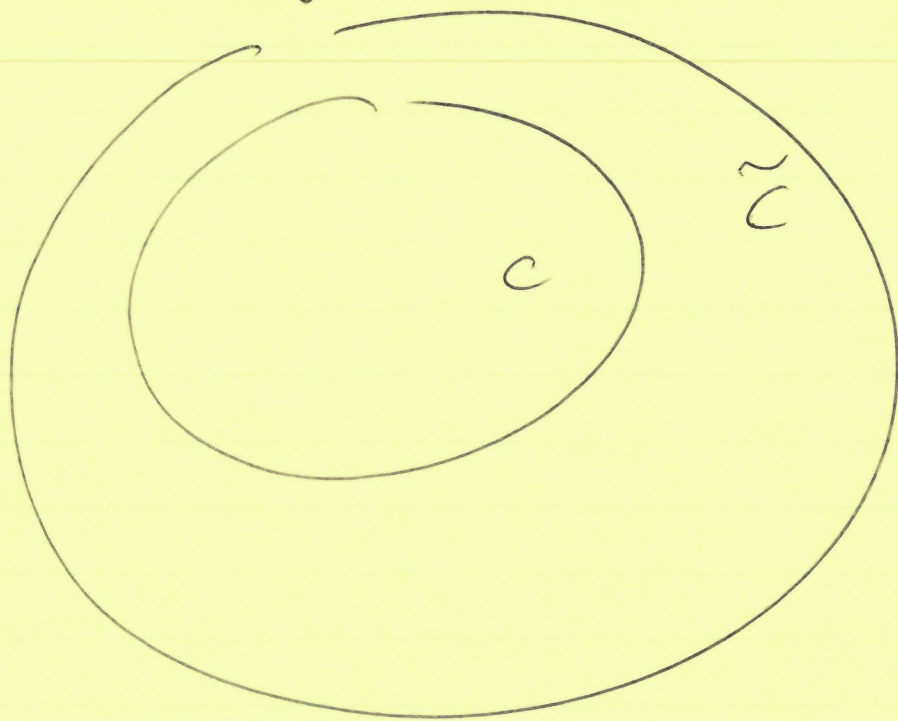
i.e.

$My \in \text{null}(A)$  for all  $y$   
 $AMy = 0$

i.e.

$$x = My + x_0 \iff Ax = b.$$

some  $y$



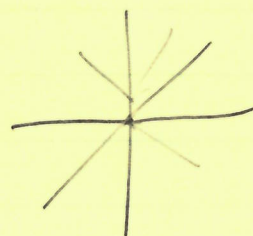
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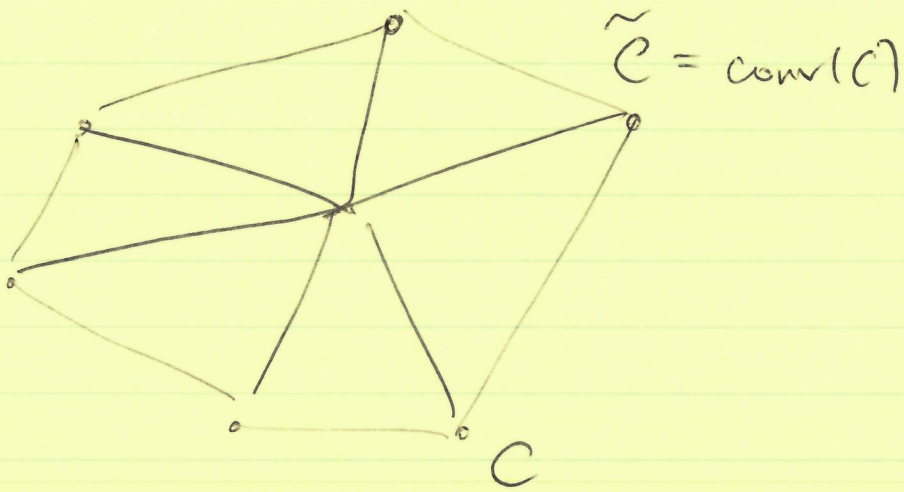
$C = \{R: \text{rank}(R) = k\}$  is not convex

$$A \in C, -A \in C$$

$$\frac{1}{2}A + \frac{1}{2}(-A) = 0 \notin C$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$





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$$XZ = XVV^T$$

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• Convex  
problems

nonConvex  
problems

$$Dx \geq d$$

rows of  $D$ :  $D_i$

$$D_i^T x \geq d_i \quad \text{for all } i.$$

$$\begin{aligned} \min \quad & \|\beta\|_1 \\ \text{st.} \quad & |x_j^T (y - X\beta)| \leq \lambda \\ & \text{for all } j=1, \dots, p \end{aligned}$$

( $x_j$  is  $j^{\text{th}}$  column of  $X$ )

why not  $|y_i - x_i^T \beta| \leq \lambda$  ?  
 all  $i=1, \dots, n$   
 $x_i$  is  $i^{\text{th}}$  row of  $X$

$$\text{Min}_{\beta} \|y - X\beta\|_2^2$$

$$\Leftrightarrow 2X^T(X\beta - y) = 0$$

$$\text{ie } \underline{X^T(y - X\beta) = 0}$$

change  $\leq$  on vectors to

$\leq$  on matrices

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$$a_i^T x = b_i \quad \left. \vphantom{a_i^T x = b_i} \right\} \quad A_i \circ x = b_i$$

$$\begin{aligned} \text{min} \quad & c^T x \\ \text{st.} \quad & Gx \leq h \quad \Leftrightarrow \sum_i G_i x_i \leq h \\ & Ax = b \end{aligned}$$

$G_i$  is  $i^{\text{th}}$  column  
of  $G$

LPs use

$$G_1 x_1 + \dots + G_n x_n \leq h$$

SDPs use

$$F_1 x_1 + \dots + F_n x_n \preceq F_0$$

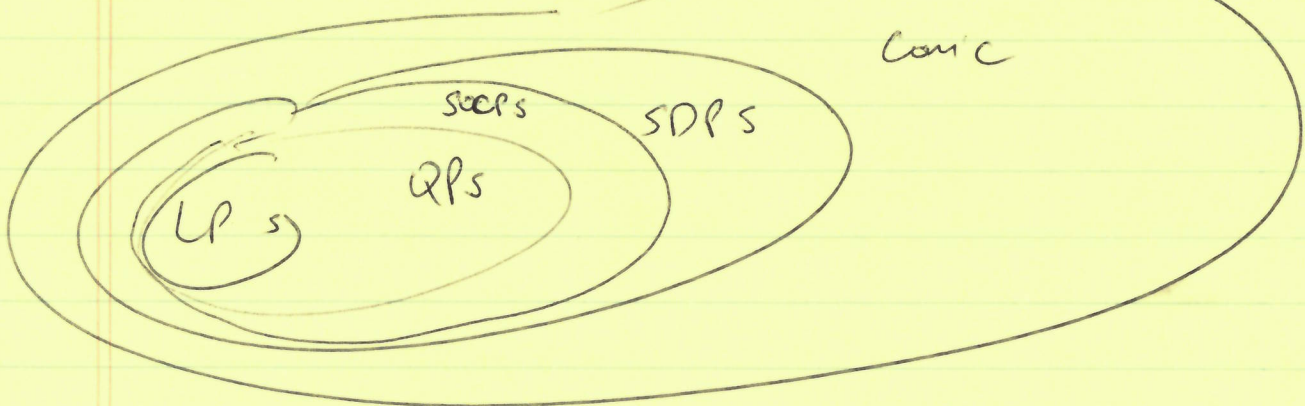
$$\begin{aligned} Dx + d &\in \mathbb{R}_+^n \\ \Leftrightarrow Dx + d &\geq 0 \end{aligned}$$

LPs

$$\frac{1}{2} x^T Q x \leq t$$

$$\Leftrightarrow \frac{1}{2} x^T Q x \leq t$$

Conic



$w^T w$