

vectors  $x, y$   
 $x \leq y$  means  $x_i \leq y_i$  for all  $i$

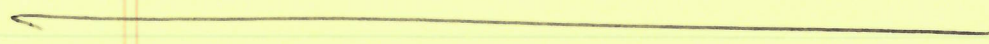
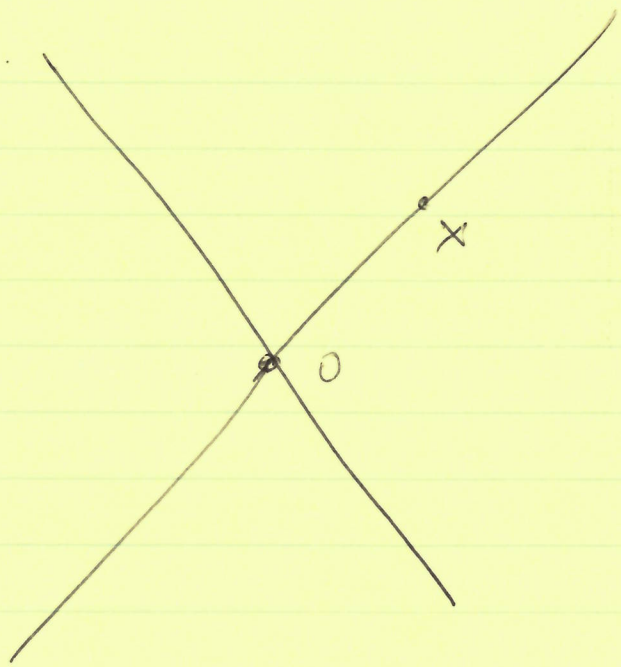
$$\{x: Ax \leq b\} = \left\{x: \begin{array}{c} a_i^T x \leq b_i \\ \uparrow \\ \text{all } i \\ \text{ith row of } A \end{array} \right\}$$

$$Cx = d \quad \begin{array}{l} Cx \leq d \\ Cx \geq d \end{array} \Leftrightarrow -Cx \leq -d$$

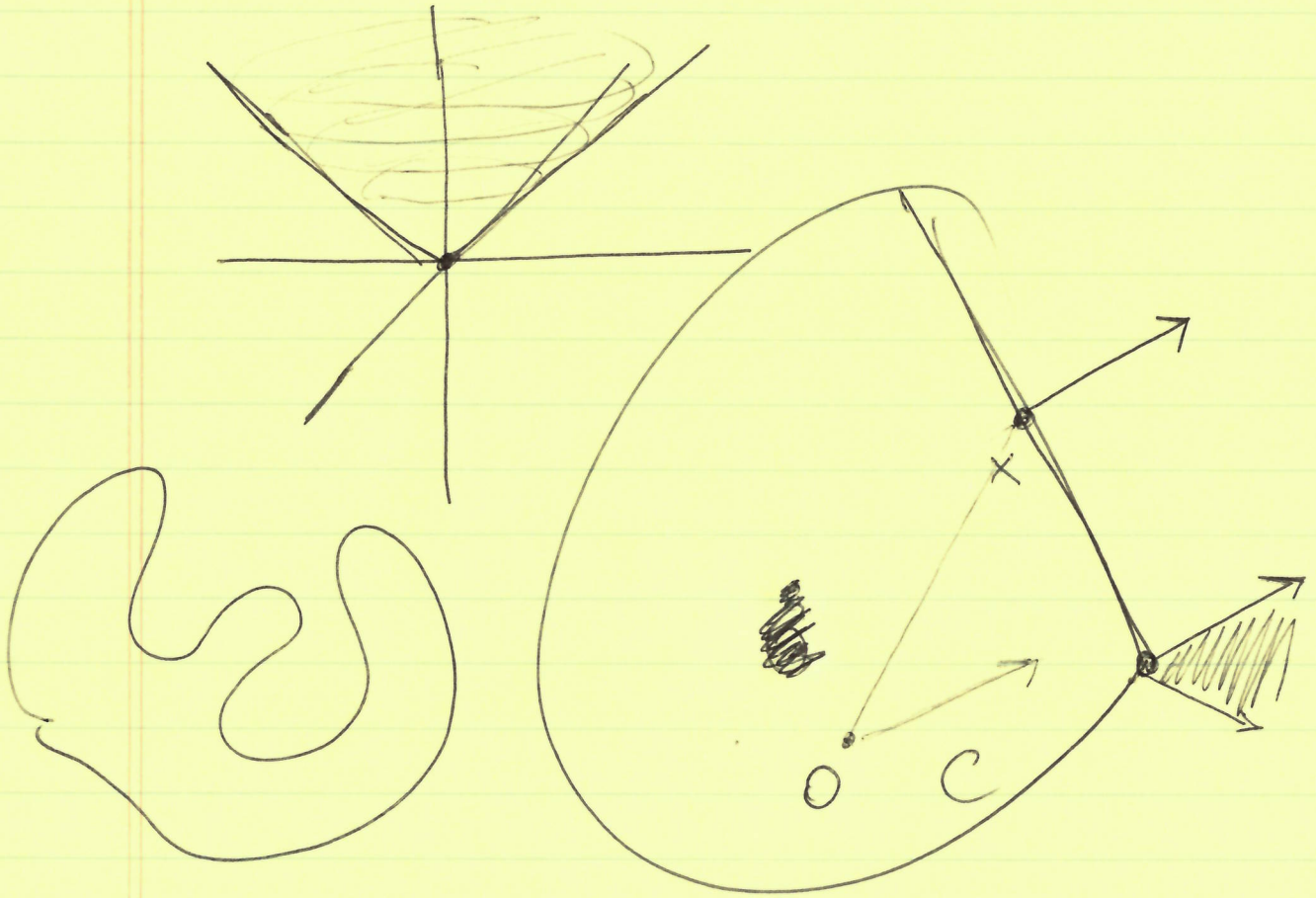
$x_1, \dots, x_k$  linearly ind:  
 means  $a_1 x_1 + \dots + a_k x_k = 0$   
 $\Rightarrow a_1 = \dots = a_k = 0$

$x_0, x_1, \dots, x_k$  affinely ind:  
 means  $x_1 - x_0, \dots, x_k - x_0$   
 are linearly ind.

2.



$$\{(x, t) : \|x\|_2 \leq t\}$$



$$N_C(x) = \{g : g^T x \geq g^T y \text{ all } y \in C\}$$

$$* g_1 \in N_C(x), g_2 \in N_C(x)$$

$$t_1 g_1 + t_2 g_2 \in N_C(x) \quad ? \quad \text{all } t_1, t_2 \geq 0.$$

$$\begin{aligned} & (t_1 g_1 + t_2 g_2)^T x \\ &= t_1 g_1^T x + t_2 g_2^T x \\ & \text{know that } g_1^T x \geq g_1^T y \text{ all } y \\ & \quad \quad \quad g_2^T x \geq g_2^T y \text{ all } y \\ & \geq t_1 g_1^T y + t_2 g_2^T y \text{ all } y \\ &= (t_1 g_1 + t_2 g_2)^T y \end{aligned}$$

$X \succeq 0$  means that  $(X \in S^n)$

all eigenvals of  $X$  are  $\geq 0$

i.e.

$$a^T X a \geq 0 \text{ all } a \in \mathbb{R}^n$$

$$X, Y \in S_+^n$$

$$t_1 X + t_2 Y \in S_+^n \quad ? \quad \text{all } t_1, t_2 \geq 0$$

$$a^T (t_1 X + t_2 Y) a = t_1 a^T X a + t_2 a^T Y a$$

$$\geq 0 \quad \text{all } a \in \mathbb{R}^n$$

4.

$$C = \{a^T x \leq b\}$$

$$D = \{a^T x > b\}$$

can't

 $\tilde{a}, \tilde{b}$ 

st.

$$\tilde{a}^T x < \tilde{b} \text{ on } C$$

$$\tilde{a}^T x > \tilde{b} \text{ on } D$$

$$x_1 A_1 + \dots + x_k A_k \preceq B \text{ means}$$

$$B - (x_1 A_1 + \dots + x_k A_k) \succeq 0$$

$$X \preceq Y$$

means

$$Y - X \succeq 0.$$

$$\left\{ x : \sum_{i=1}^k x_i A_i \preceq B \right\}$$

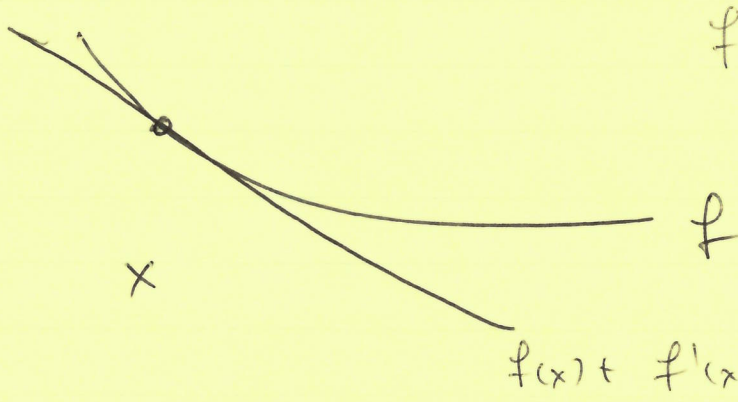
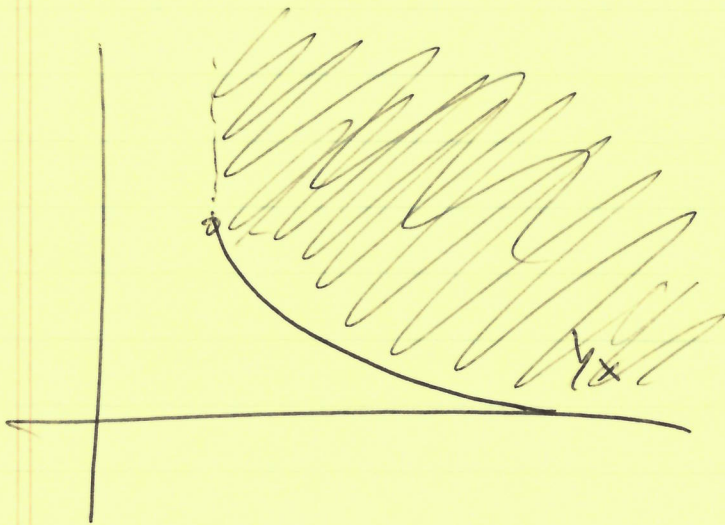
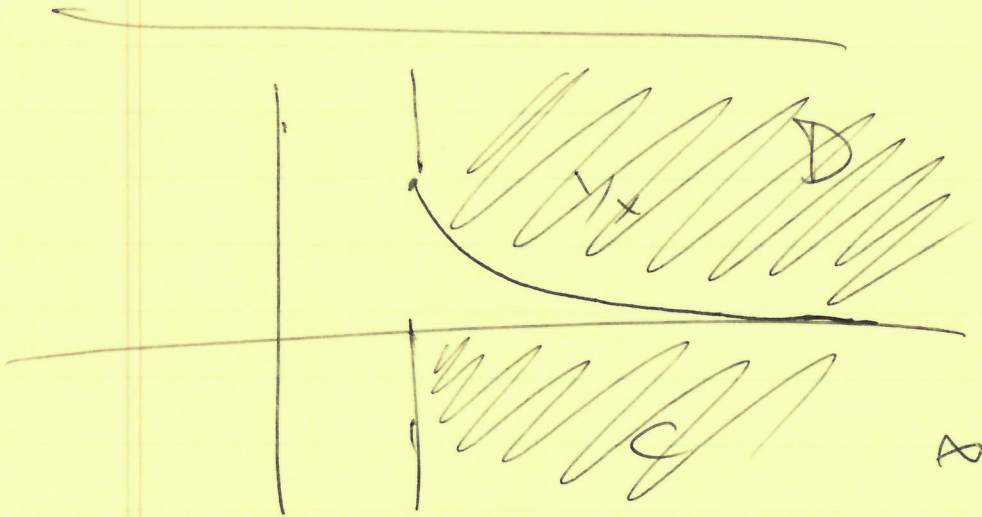
convex!

$$= \{x : \text{~~some~~ } f(x) \succeq 0\}$$

$$= f^{-1}(S_+^n)$$

$m$ -strongly convex  $f$  means  
 $g$  is convex. where

$$g(x) = f(x) - \frac{m}{2} \|x\|_2^2$$



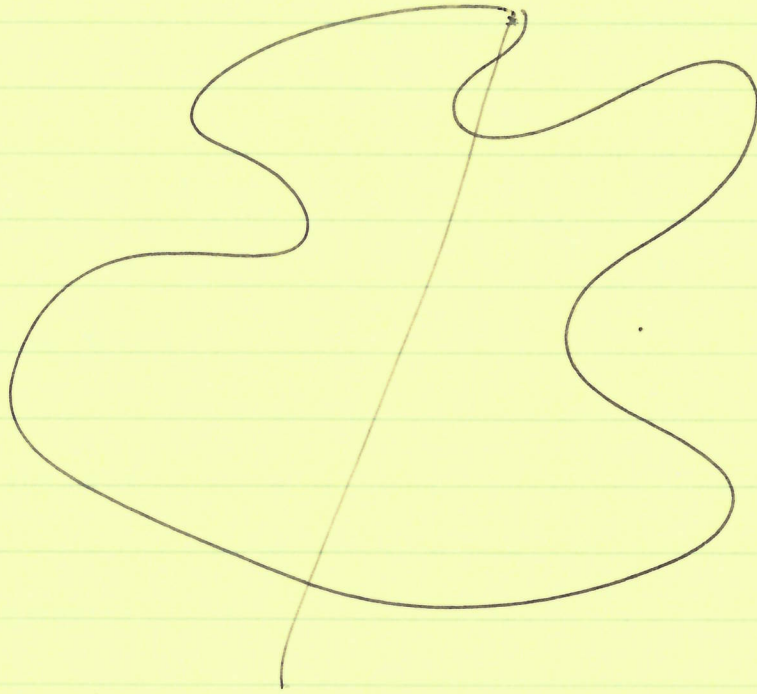
$$f''(x) \geq 0$$

all  $x$

$$[\nabla^2 f(x)]_{ij}$$

$$= \frac{\partial^2 f(x)}{\partial x_i \partial x_j}$$

6



$$f(x) = \max_{y \in C} \|y - x\|$$