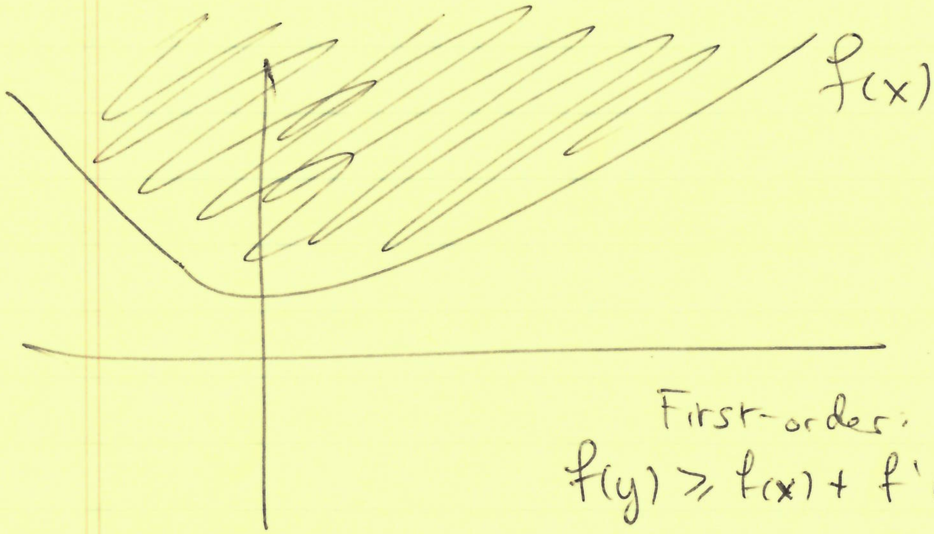
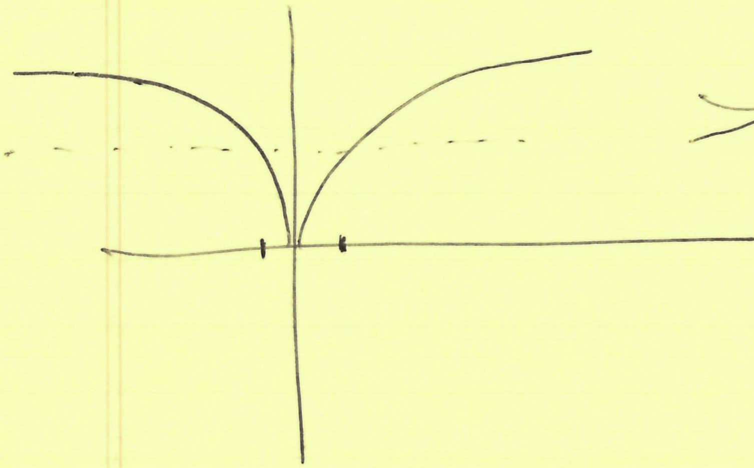
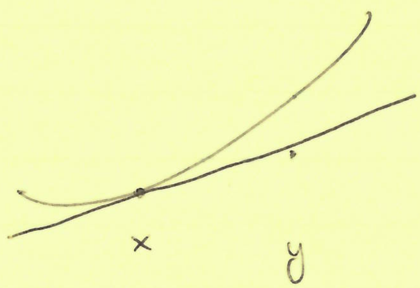
1



First-order:

$$f(y) \geq f(x) + f'(x)\cdot(y-x)$$





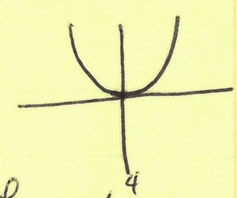
Second-order

$$f''(x) \geq 0$$

$$\underset{\text{all } x}{\nabla^2 f(x) \succ 0} \Rightarrow f \text{ strictly convex}$$

$\not\Leftarrow$

this direction doesn't hold

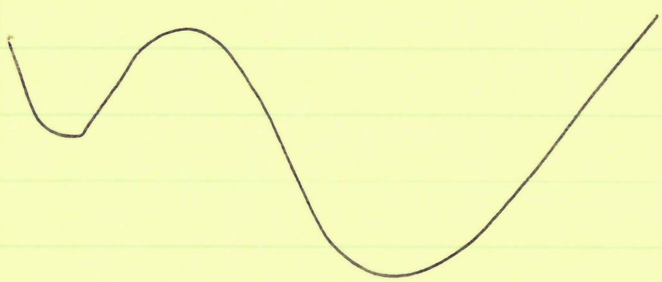


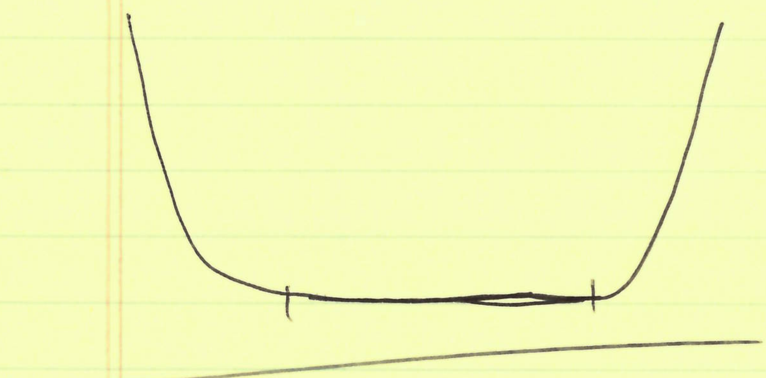
$f(x) = x^4$

strictly convex

$$\max\{-7\log(a^Tx+b),\ \|Ax+b\|_1^5\}$$

$$\max\{\underline{-7\log \cancel{x}y},\ \underline{\|x\|_1^5}\}$$





Lasso:

$g_1(\beta) = \|\beta\|_1 - s$

$\underset{n\times p}{X}$, $\underset{p\times p}{X^TX}$ $\qquad X^TX \succ 0 \iff X^TX$ invertible

$\iff X$ having linearly indep. columns

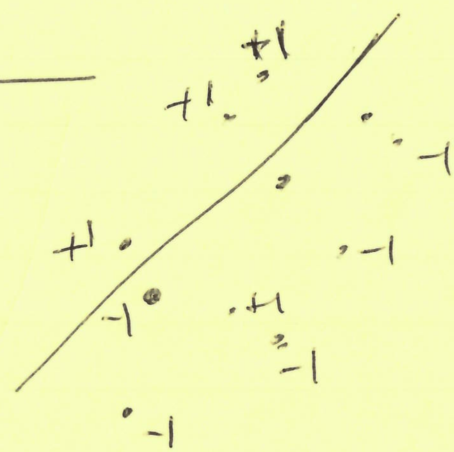
$n<p$. $\quad X^T X$ is singular

$\qquad\; p\times p \quad$ ie non-invertible

$\|y - X\beta\|_2^2 = f(\beta)$

$= \beta^T X^T X\beta - 2y^T X\beta + y^T y$

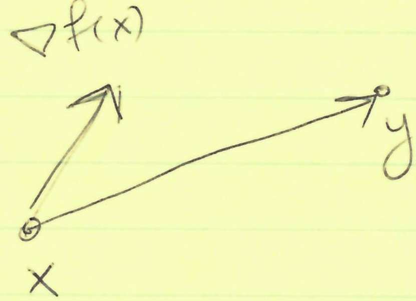
take $\beta \neq 0$ with $X\beta = 0$



$-\xi_i \le 0$

$-y_i^T(x_i^T\beta+\beta_0) + 1 - \xi_i \le 0$

$$C = \bigcap_{i=1}^{m} \{x : g_i(x) \le 0\} \cap \{x : Ax = b\}$$



$bx + c$

SVMs

$\xi_i \geq 0, \quad y_i(x_i^T \beta + \beta_0) \geq 1 - \xi_i$

$$\Longleftrightarrow \quad \begin{aligned} 1 - y_i(x_i^T\beta + \beta_0) &\leq \xi_i \\ 0 &\leq \xi_i \end{aligned}$$