①

$$f^\star > g^\star$$

$$(g^\star)^\star \geq g^\star$$

$$\max \; g(u,v)$$
$$\text{s.t.} \quad u \geq 0$$

$\Longrightarrow$

$$g(u,v) = \min_x L(x,u,v)$$
$$= \min_x f(x) + \sum u_i h_i(x) + \sum v_j l_j(x)$$

$$\min_x L(x, u^\star, v^\star) \leq L(x^\star, u^\star, v^\star)$$

$$g(u^\star, v^\star) = \min_x L(x, u^\star, v^\star)$$
$$= L(x^\star, u^\star, v^\star) \quad \text{(stationarity)}$$

$$g(u^\star, v^\star) = \min_x L(x, u^\star, v^\star) = L(x^\star, u^\star, v^\star)$$

②

$$L(x,u) = \frac{1}{2}x^TQx + c^Tx + u^TAx$$

Stationarity.

$$Qx + c + A^Tu = 0$$

comp. slackness:

$$\emptyset$$

primal & dual feasibility

$$Ax = 0$$

$$\Longleftrightarrow \quad \underbrace{\begin{bmatrix} Q & A^T \\ A & 0 \end{bmatrix}}_{\text{"KKT" matrix}} \begin{bmatrix} x \\ u \end{bmatrix} = \begin{bmatrix} -c \\ 0 \end{bmatrix}$$

$$\min_d \quad \frac{1}{2}d^TQd + c^Td$$

$$A\,d = 0$$

$$Q = \nabla^2 f(x^{(k-1)})$$
$$c = \nabla f(x^{(k-1)})$$

$$\min \quad f(x)$$
$$\text{s.t.} \quad Ax = b.$$

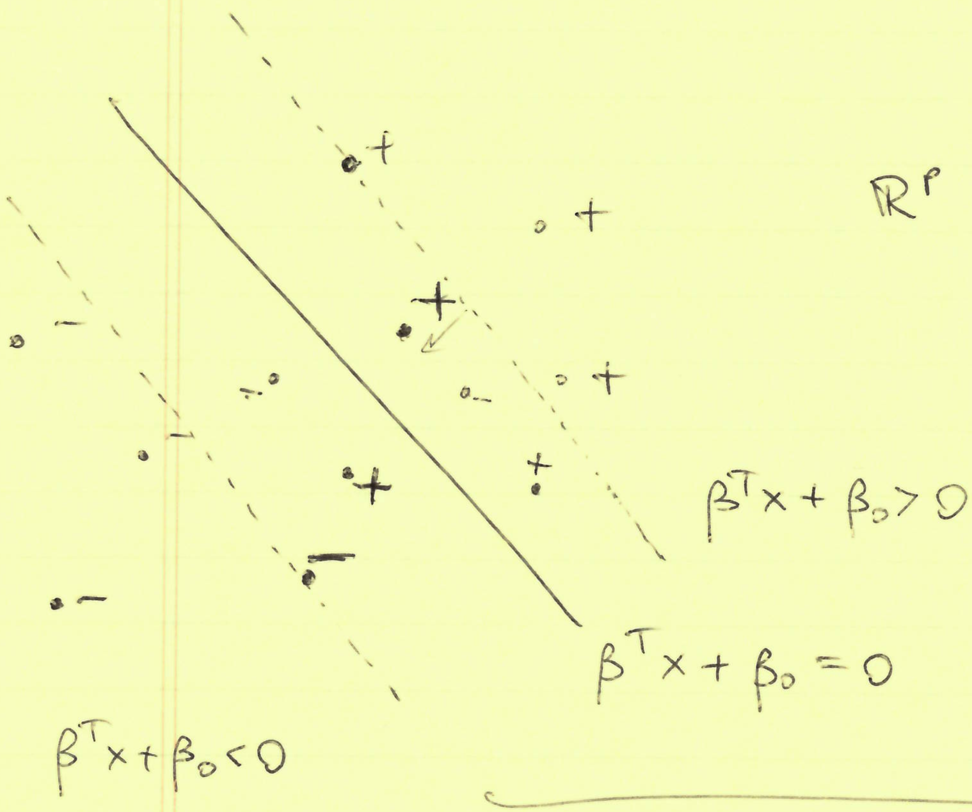
start with $x^{(0)}$: $Ax^{(0)} = b$

$$L(x,u,v) = -\sum \log(\alpha_i + x_i) - \sum u_i x_i + v(1^Tx - 1)$$

(3)

stationarity:

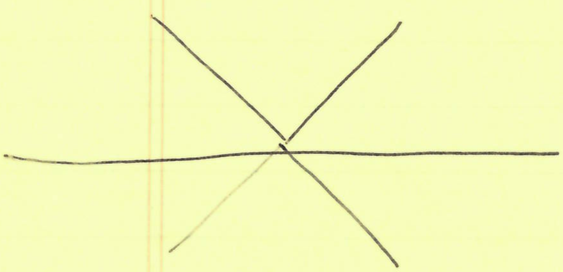
$$0 \in \partial \Big(\underbrace{f(x) + \sum u_i h_i(x) + \sum v_j \ell_j(x)}_{L(x, u, v)} \Big)$$



$L(\beta, \beta_0, \xi, w, v) =$

$\frac{1}{2}\|\beta\|_2^2 + C \sum \xi_i \cancel{*} - \sum v_i \xi_i$

$+ \sum w_i (1 - \xi_i - y_i(x_i^T \beta + \beta_0))$



$\beta - \sum w_i y_i x_i = 0$

$\sum w_i y_i = 0$

$C - v_i - w_i = 0, \quad i = 1, \ldots n$