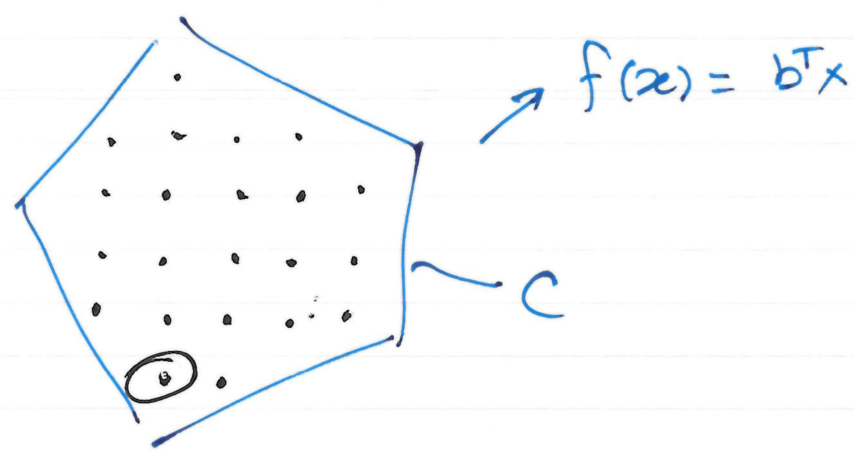
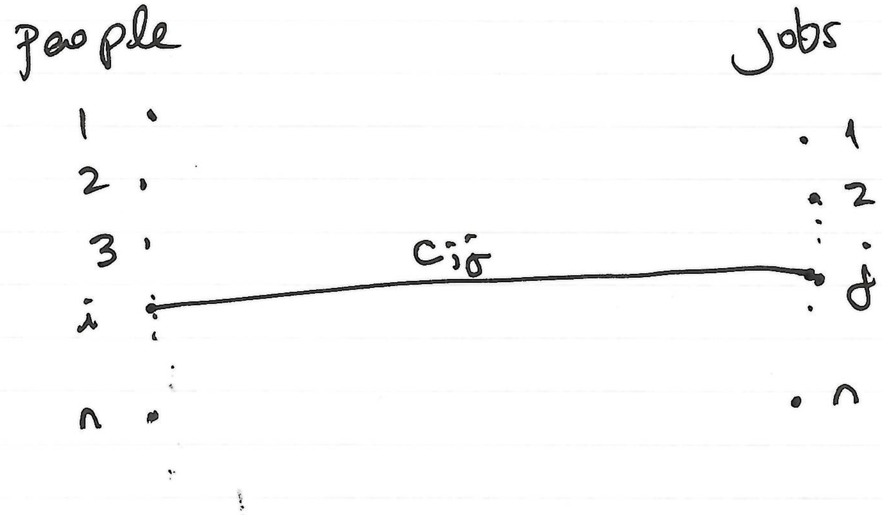
Integer program



Assignment problem:



IP formulation for K-medoids

$$d_{ij} := \|x^{(i)} - x^{(j)}\|^2$$

$$w_i = \begin{cases} 1 & \text{if choose } x^{(i)} \text{ as a centroid} \\ 0 & \text{o.w} \end{cases}$$

$$z_{ji} = \begin{cases} 1 & \text{if } x^{(j)} \text{ is in the cluster w/ centroid } x^{(i)} \\ 0 & \text{o.w} \end{cases}$$

$$\min_{w,z} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} z_{ji}$$

$$z_{ji} \leq w_i \quad i,j = 1 \ldots n$$

$$\sum_{i=1}^{n} w_i = k$$

$$w_i \in \{0,1\} \quad i = 1 \ldots n$$

$$z_{ji} \in \{0,1\} \quad j,i = 1 \ldots n$$

IP formulation for best subset selection

$$\min_{\beta, z} \|y - X\beta\|_2^2$$

$$|\beta_i| \leq M z_i$$

$$z_i \in \{0,1\}, \quad i = 1, \ldots, p$$

$$\sum_{i=1}^{p} z_i \leq k$$

M: a priori bound on entries of β.

$$z = \min_x f(x) \quad x \in X$$

Suppose $\bar{z} \geq z$ & $\underline{z} \leq z$

and $\underline{z} \leq f(x^*) \leq \bar{z}$

$\Rightarrow \quad 0 \leq f(x^*) - z \leq \bar{z} - \underline{z}$

Observe: if $\min_{x \in Y} g(x)$ is a relaxation of $\min_{x \in X} f(x)$

then

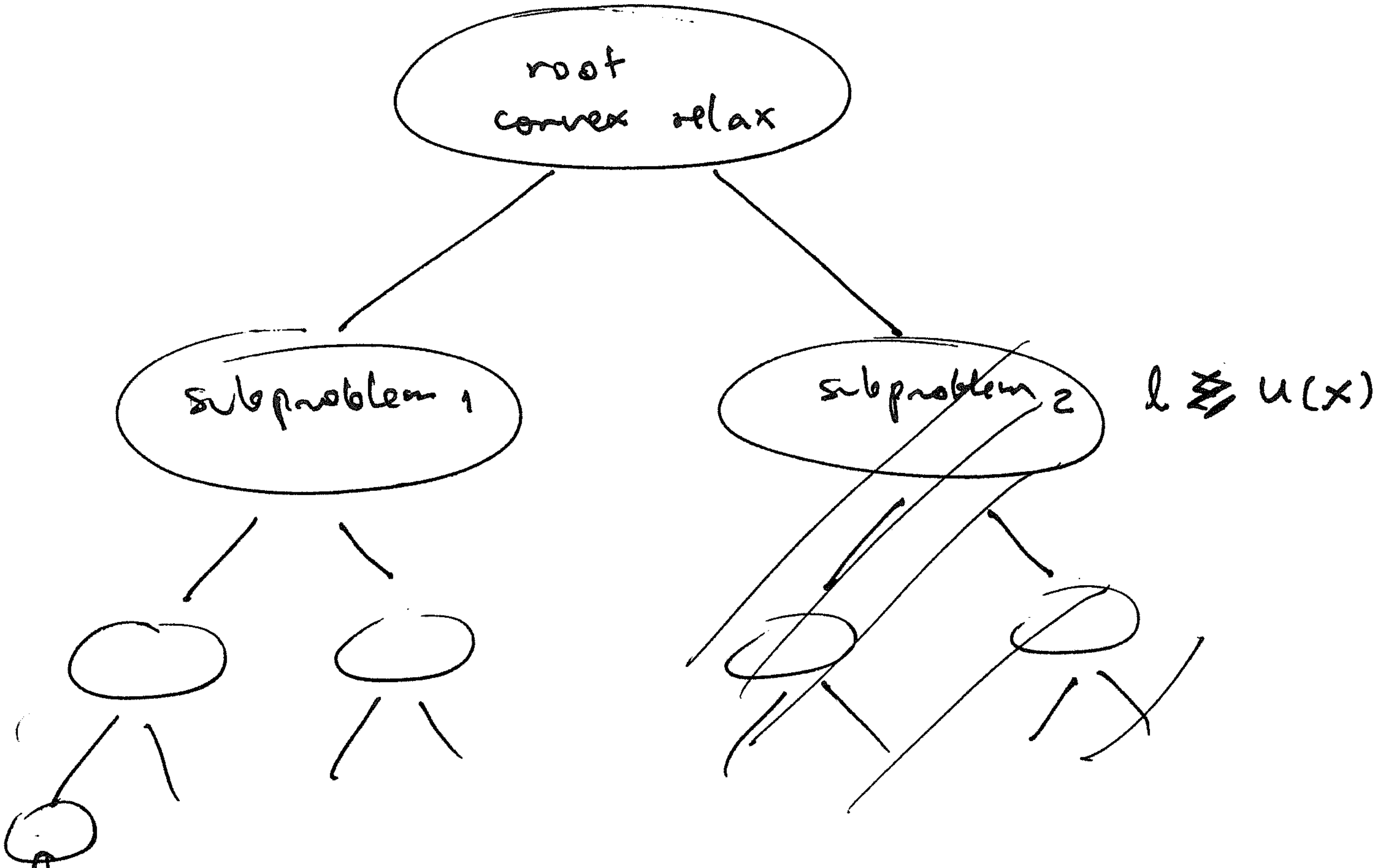
$Y \subseteq X$ $\qquad$ $x \in X \Rightarrow g(x) \leq f(x)$

$$\min_{x \in Y} g(x) \leq \min_{x \in X} g(x) \leq \min_{x \in X} f(x)$$

Knapsack:
$$\max_x \; c^T x$$
$$a^T x \leq b$$
$$x_j \in \{0,1\} \quad j = 1, \dots n$$

LP-relaxation
$$\max \; c^T x$$
$$a^T x \leq b$$
$$0 \leq x \leq 1$$

Typical B&B tree



feas sln to → $u(x)$
(IP)