Recall : If $r : \mathbb{R}^d \to \mathbb{R}^d$

Newton's method for $r(w) = 0$

$$w^+ = w - r'(w)^{-1} r(w)$$

Claim: $\frac{1}{2}\|r(w)\|^2 = f(w)$

$\to$ $\Delta w = -r'(w)^{-1} r(w)$ is a descent direction for $f(w)$

Δw is also a descent direction for $\|r(w)\|$

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① Observe:

$$\min \; c^T x - \tau \sum \log(x_i)$$
$$Ax = b$$

KKT condns : $A^T y + \tau X^{-1} \mathbb{1} = c$

$Ax = b$

$$\max \; b^T y + \tau \sum \log(s_i)$$
$$A^T y + s = c$$

KKT condns : $\begin{bmatrix} b \\ -\tau S^{-1} \mathbb{1} \end{bmatrix} = \begin{bmatrix} A \\ I \end{bmatrix} v$ & $A^T y + s = c$

$\iff$

$$\tau A S^{-1} \mathbb{1} = b$$
$$A^T y + s = c$$