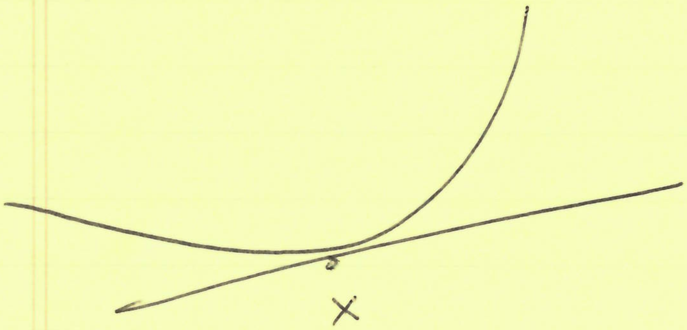
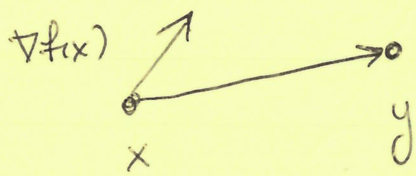
$x \in C \qquad y \notin C$

$I_C(x) \qquad I_C(y)$

$0 \qquad \infty$



- Always exists (on the relative interior of the domain)



$A+B = \{a+b : a \in A, b \in B\}$

$0 \in \partial f(x) + N_C(x)$

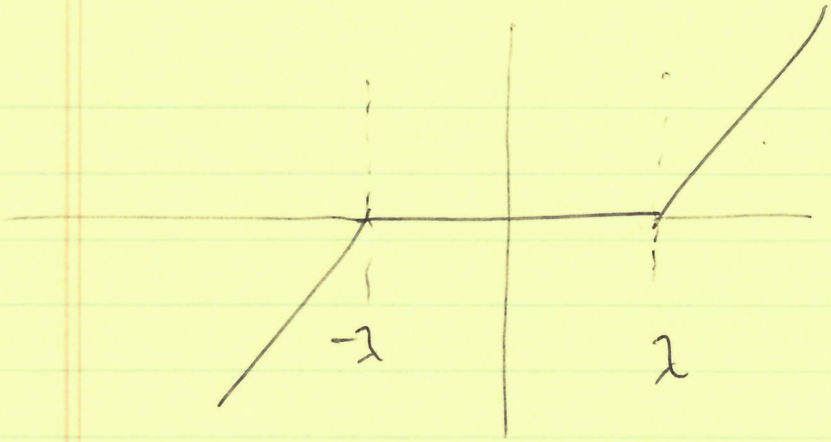
$$C = \left\{ x : \begin{array}{l} g_i(x) \le 0 , \ i=1 \ldots m \\ Ax = b \end{array} \right\}$$

$X = [X_1 \cdots X_{100}] \ [X_{101} \cdots X_{200}]$

$\beta_{1:100}$

$|X_j^T (y - X_{1:100}\beta_{1:100})| \le \lambda$ for all $j = 101, \ldots 200$

$S_\lambda(x)$



$$\beta_i = S_\lambda(y_i)$$

- if $y_i > \lambda$, then $\beta_i = y_i - \lambda > 0$

  $\text{sign}(\beta_i) = 1$

  $y_i - \beta_i = \lambda \cdot \text{sign}(\beta_i)$ ✓

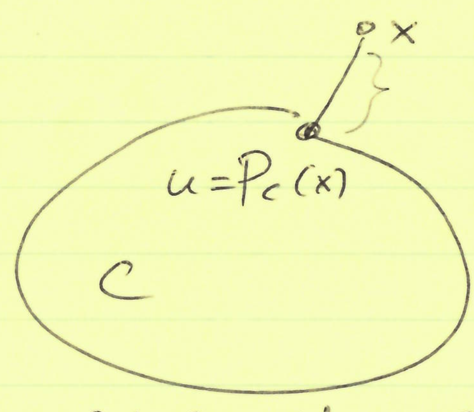
- if $y_i < -\lambda$, then ... ✓

- if $-\lambda \le y_i \le \lambda$. then $\beta_i = 0$

  $y_i - \beta_i = y_i \in [-\lambda, \lambda]$ ✓

$$P_C(x) = \underset{y \in C}{\text{argmin}} \; \|y - x\|_2$$

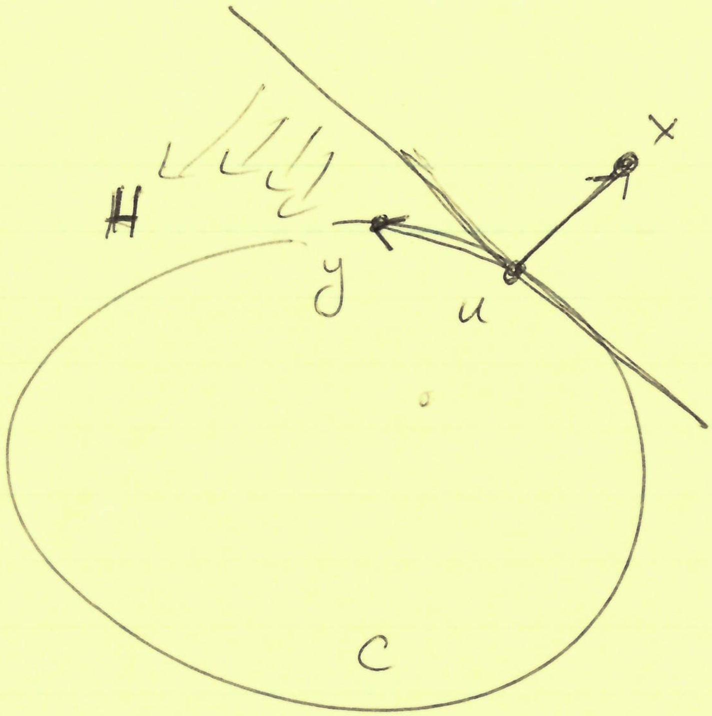
$$\text{dist}(x, C) = \|x - P_C(x)\|_2$$



$$\min_{y \in C} \tfrac{1}{2}\|y - x\|_2^2$$

$u = P_C(x)$

$(u - x)^T(y - u) \ge 0$ all $y \in C$



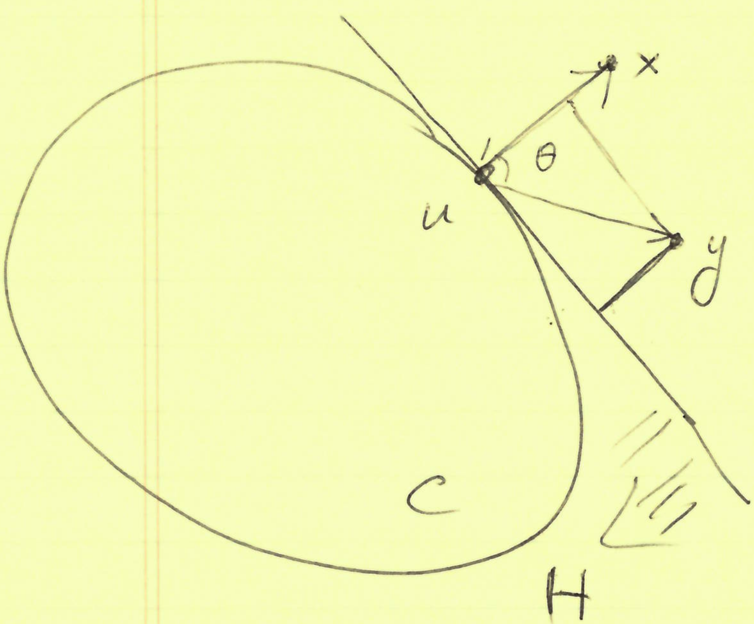
$(u-x)^T(y-u) \geq 0$ all $y \in C$

$\iff$

$(x-u)^T(y-u) \leq 0$ all $y \in C$

$$\text{dist}(y,C) \geq \frac{(x-u)^T(y-u)}{\|x-u\|_2} \quad \text{all } y$$

- ~~if y∈H~~ if $y \in H$, then RHS is negative ✓
- if $y \notin H$, then picture works ✓



~~all y~~

$(x-u)^T(y-u)$
$= \|x-u\|_2 \cdot \cos\theta \cdot \|y-u\|_2$

4

Square summable but not summable

eg. $t_k = 1/k$

$\varepsilon = 10^{-3}$