

①

$$A \begin{array}{|c|} \hline \text{---} \\ \text{---} \\ \text{---} \\ \hline \end{array} \left| \begin{array}{c} b \\ \parallel \\ n \end{array} \right. \quad m \quad n$$

$$\begin{array}{|c|} \hline \text{---} \\ \text{---} \\ \text{---} \\ \hline \end{array} \quad m \quad \begin{array}{c} P \\ \parallel \\ n \end{array}$$



$$\nabla^2 f(x) v = \nabla f(x) \\ Av = b.$$

$$A = \begin{bmatrix} a_1 & \dots \\ \vdots & \ddots a_n \end{bmatrix}$$

$$A = \begin{bmatrix} \text{wavy lines} & 0 \\ 0 & \text{wavy lines} \end{bmatrix}$$

$$Ax = b, Ax = c, Ax = d \dots$$

$$n^3 + 3n^2.$$

A \ B

$$Ax = b$$

$$QRx = b$$

$$Rx = Q^T b.$$

$$x = R^{-1} Q^T b \quad n^2$$

$$A = B^T B.$$

$$B = QR$$

$$B^T B = R^T Q^T Q R$$

$$= R^T R$$

$$:= L L^T.$$

(2)

$$Ax = b$$

$$\underbrace{LL^T}_{y}x = b.$$

$$Ly = b. \quad n^2 \text{ forward substitution.}$$

$$y = L^{-1}b.$$

$$L^Tx = y$$

$$x = (L^T)^{-1}y \quad n^2 \text{ back substitution}$$

$$x^T x \beta = x^T y \quad \text{"normal equations".}$$

$$n \times p \quad Q \text{ from QR}$$

$$n \times (n-p) \quad \tilde{Q} \text{ orthogonal completes the basis}$$

$$P = [Q \ \tilde{Q}] \text{ orthogonal}$$

$n \times n$

$$\begin{aligned} \|x\|_2^2 &= \|P^T x\|_2^2 = \left\| \begin{bmatrix} Q^T \\ \tilde{Q}^T \end{bmatrix} x \right\|_2^2 = \left\| \begin{bmatrix} Q^T x \\ \tilde{Q}^T x \end{bmatrix} \right\|_2^2 \\ &= \|Q^T x\|_2^2 + \|\tilde{Q}^T x\|_2^2. \end{aligned}$$

$$x = y - x\beta. \quad \text{write } x = QR$$

$$\begin{aligned} \|y - x\beta\|_2^2 &= \|Q^T(y - QR\beta)\|_2^2 \\ &\quad + \|\tilde{Q}^T(y - QR\beta)\|_2^2 \end{aligned}$$

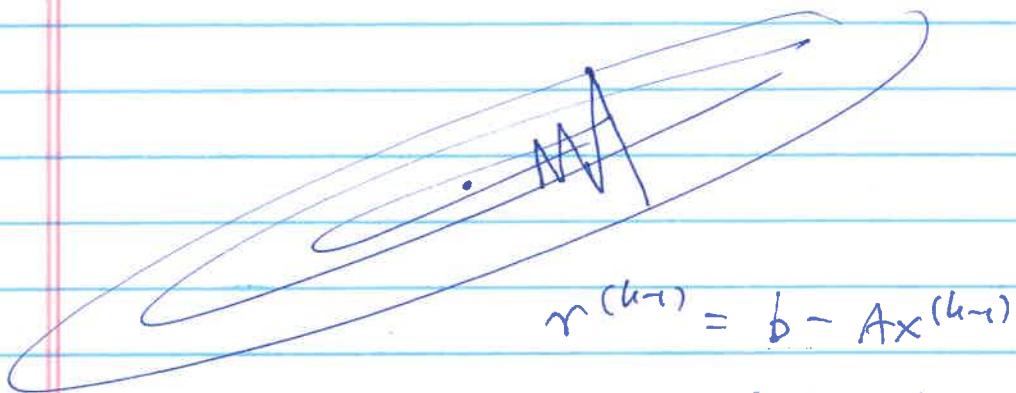
$$= \|Q^T y - R\beta\|_2^2 + \|\tilde{Q}^T y\|_2^2$$

$$R\beta = Q^T y \quad \text{solve: } \underset{p^2}{\text{back substitution}}$$

(3)

$$f(x) = \frac{x^T A x - b^T x}{2}$$

$$\min f(x) \iff \nabla f(x) = 0 \\ Ax = b.$$



$$x^{(k)} = x^{(k-1)} + t_k p^{(k-1)}$$

$$\|y - X\beta\|_2^2$$

$$x^T x \beta = x^T y$$