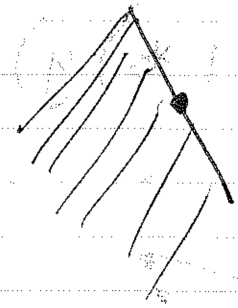
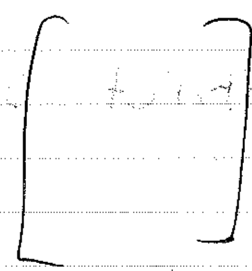


$x, y \qquad \underline{\theta x + (1-\theta)}\, y$

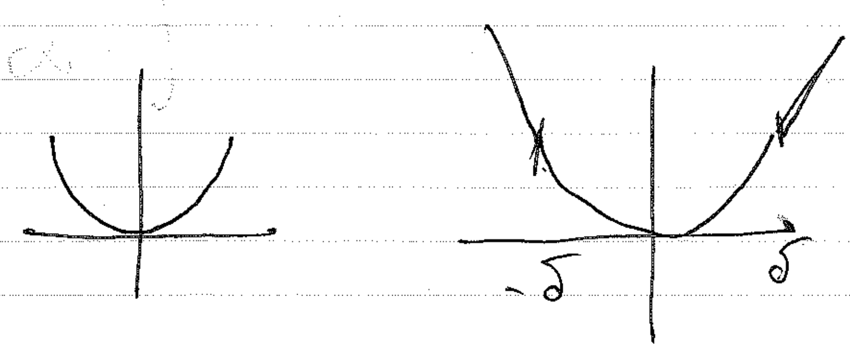
$$f(\theta x + (1-\theta) y) < \theta f(x) + (1-\theta) f(y)$$

$$f^* < f^*$$

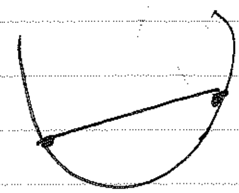


full rank

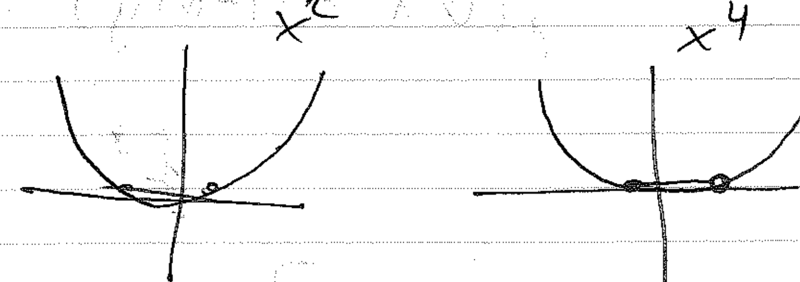
$X^T X$



$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$$

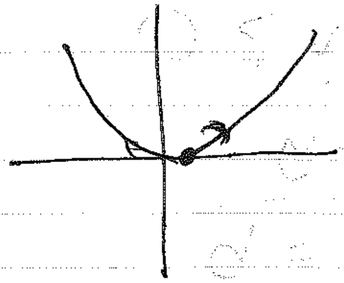


$$f(tx + (1-t)y) < tf(x) + (1-t)f(y)$$



Strong $\rightarrow$ Strict $\rightarrow$ convex

$$I_C(x) = \begin{cases} 0 & x \in C \\ \infty & \text{else} \end{cases}$$



$$\nabla f(x)^T (y-x) \geq 0$$

$$y = x \pm e_i \qquad \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\nabla f_i(x) \geq 0$$

$$-\nabla f_i(x) \geq 0$$

$$\nabla f_i(x) = 0 \quad \forall i = 1, \ldots, n$$

$$Ay = b$$

$$Ax = b$$

$$A\underbrace{(y-x)}_{v} = 0$$

$$v \in \text{null}(A)$$

$\underline{\nabla f(x)^T v} \geq 0$

$y' = 2x - y$

$A y' = b$

$(y' - x) = -v$

$-\underline{\nabla f(x)^T v} \geq 0$

$\underline{\nabla} f(x)^T v = 0 \quad v \in null(A)$

$\hookrightarrow \nabla f(x) + A^T u = 0 \quad \exists u$

$$Ax = b$$

$$Ay = b \qquad y \in C$$

$$A(y - x) = 0$$

$$\nabla f(x)^T v \geq 0 \qquad v \in null(A)$$

$$\alpha = 2x - y \qquad \alpha \in C$$

$$\alpha - x = -v$$

$$-\nabla f(x)^T \cancel{v} \geq 0 \quad , v \in null(A)$$

$$\nabla f(x)^T v = 0$$

$$\nabla f \perp null(A)$$

$$row(A) = null(A)^{\perp}$$

Strang

$$N_C(x) = \{g : g^T x \geq g^T y \quad \forall y \in C\}$$

$$g^T(x-y) \geq 0$$

$$\underbrace{(a-x)}_{\in N_C(A)}{}^T(x-y) \geq 0$$

$$\to \phi(y) = x$$

$$x \in C$$

$$Ax = b$$

$$x = My + x_0$$

$$A(My + x_0) = b$$

$$AMy + \underbrace{Ax_0}_{b} = b$$

$$AMy = 0$$

$$g_i(x) \le 0$$

$$\Updownarrow$$

$$g_i(x) + s_i = 0$$

$$s_i \ge 0$$

$$\tilde{C} = \mathrm{conv}(C)$$

$$R : \ \mathrm{rank}(R) = k$$

$$\begin{bmatrix} 1 & & 0 \\ & 0 & \\ 0 & & 0 \end{bmatrix} + \begin{bmatrix} 0 & & 0 \\ & 1 & \\ 0 & & 0 \end{bmatrix} \to \text{rank } 2$$

$$\underset{\substack{\uparrow \\ k}}{R} = X \underset{\substack{\uparrow \\ k}}{Z}$$