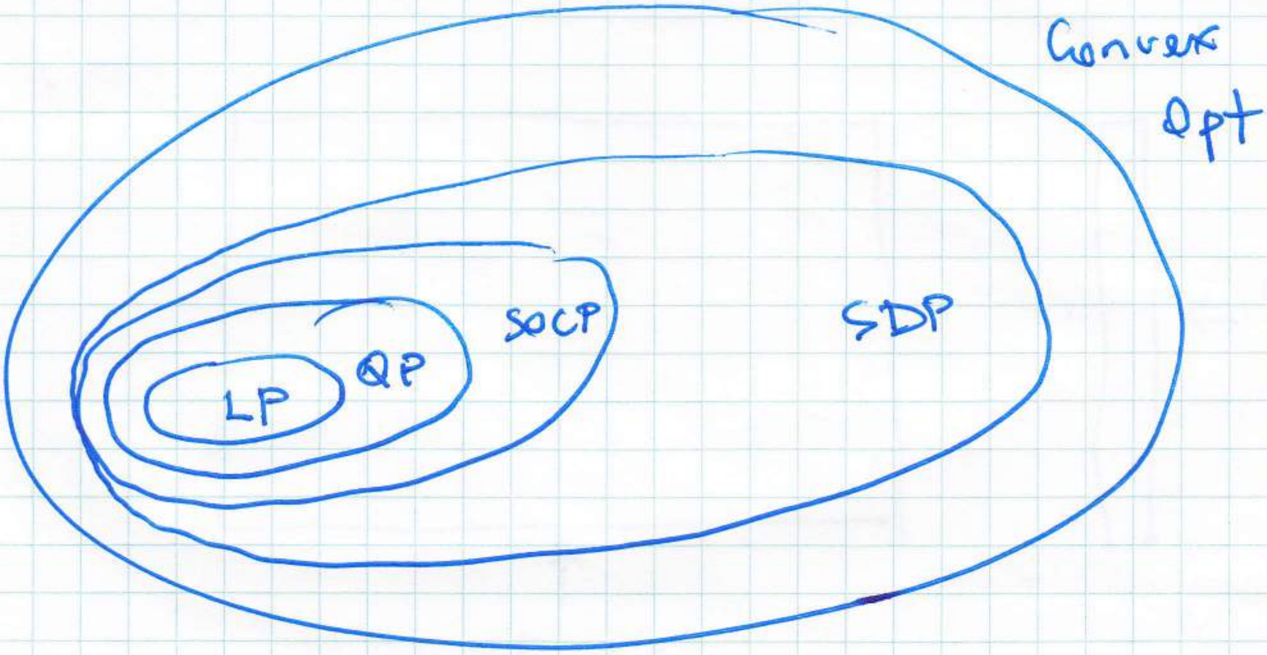


$\bar{x}$ solves

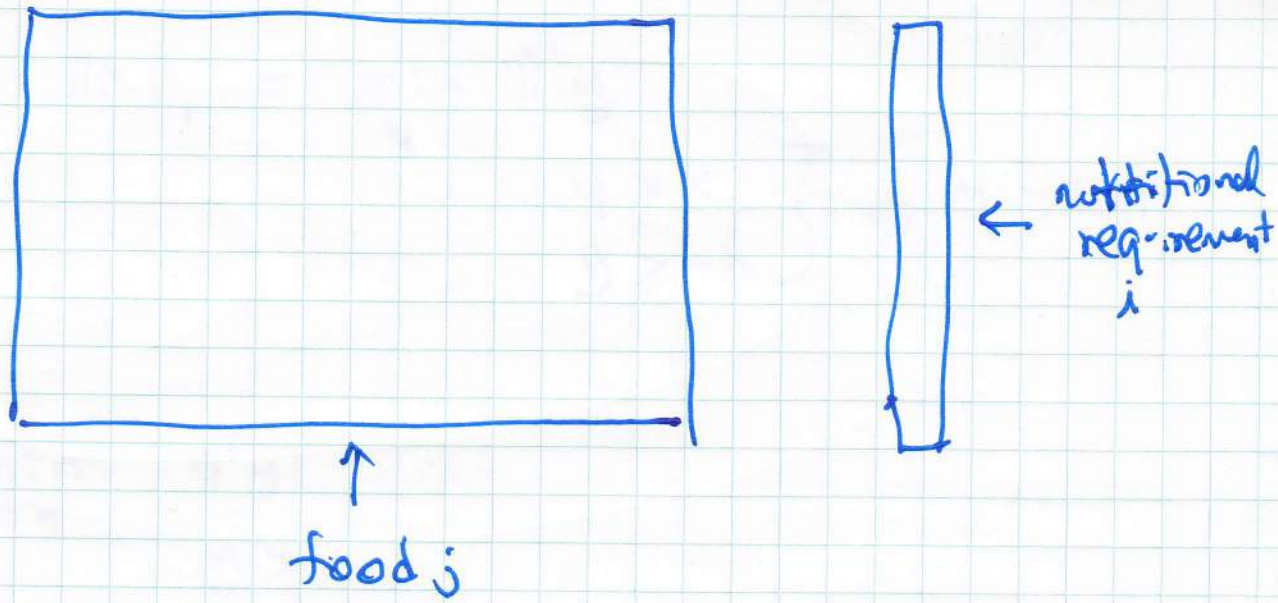
$$\min_{x \in C} f(x)$$

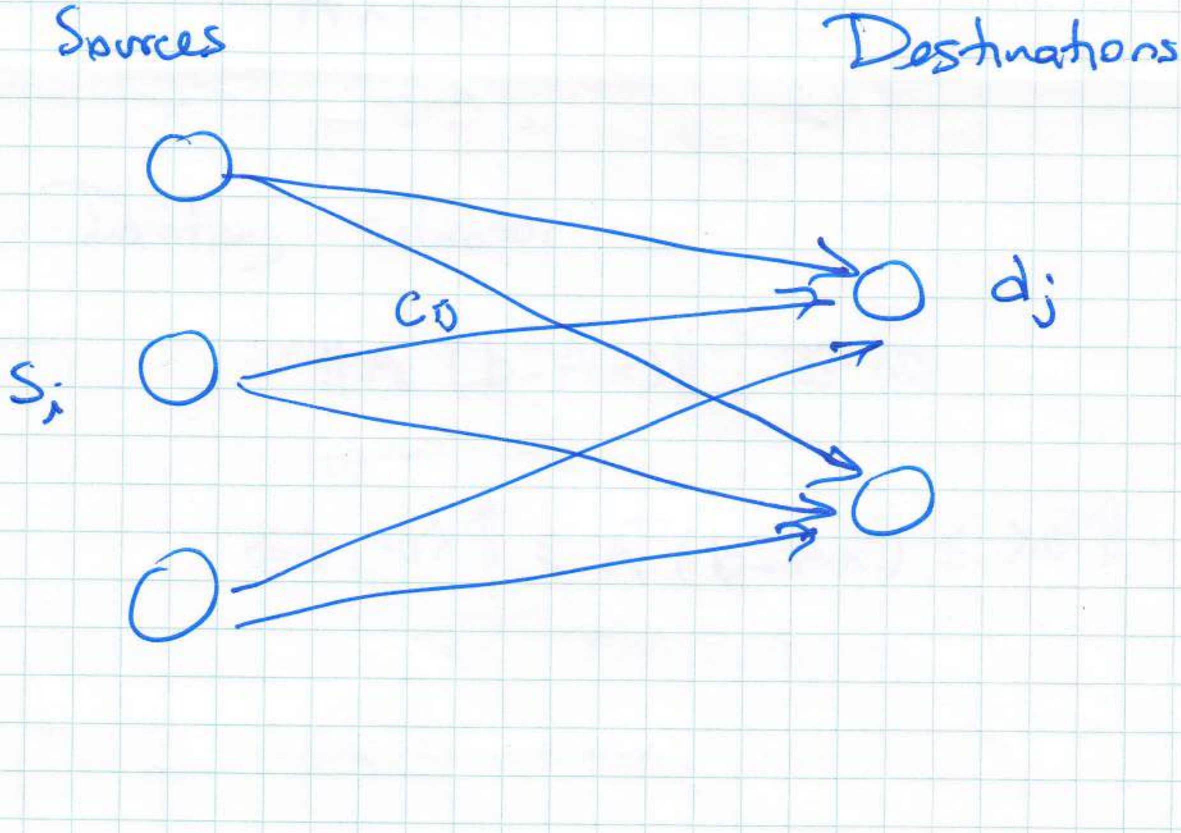
$$\iff -\nabla f(x) \in N_C(\bar{x})$$

Recall convex opt problem

$$\begin{aligned} \min \quad & f(x) \\ & g(x) \leq d \\ & Ax = b \end{aligned}$$

f, g_i are convex


nutritional
requirement
i
food j


Sources
Destinations
c_{ij}
s_i
d_j

Observe: Given x

$$\|x\|_1 = \min_y \mathbb{1}^T y$$

$$\left.\begin{array}{l} y \geq x \\ y \geq -x \end{array}\right\} \Rightarrow y_j \geq |x_j|$$

$$\min_{x,y} \mathbb{1}^T y$$

$$y \geq x$$

$$y \geq -x$$

$$Ax = b$$

In Dantzig selector:

$$\|A^T(b - Ax)\|_\infty \leq \lambda\sigma$$

$$\Leftrightarrow -\sigma\lambda \mathbb{1} \leq A^T(b - Ax) \leq \lambda\sigma \mathbb{1}$$

$$\min_{x} \quad c^T x$$
$$Dx \geq d$$

$\iff$ add slack vars

$$\min_{x,s} \quad c^T x$$
$$Dx - s = d$$
$$s \geq 0$$

$\iff$ replace $x = y - z$, $y, z \geq 0$

Rewrite the original problem as

$$\min_{y,z,s} \quad c^T (y - z)$$
$$D(y - z) - s = d$$
$$y, z, s \geq 0$$

Optimality conditions for

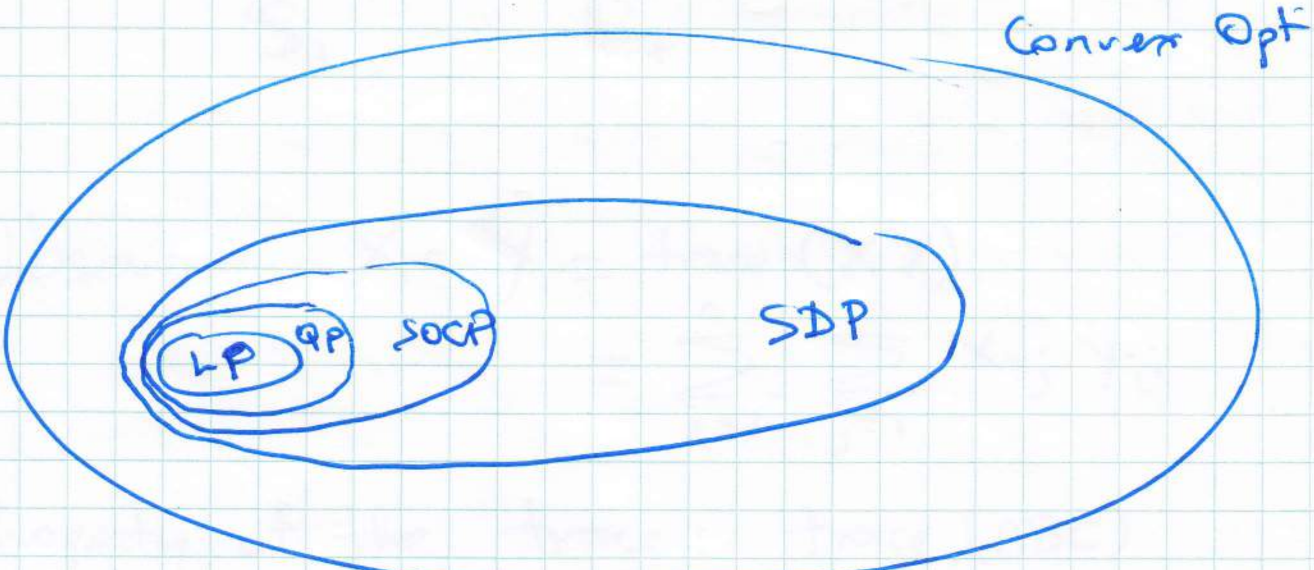
$$\min_x \; c^Tx + \frac{1}{2}x^TQx$$
$$Ax = b$$
$$x \geq 0$$

In this case $f(x) = c^Tx + \frac{1}{2}x^TQx$

Constraint set $C = \{x : Ax = b, \; x \geq 0\}$

In this case $\bar{x}$ is an optimal s/n $\iff -Q\bar{x} - c \in N_C(\bar{x})$

$\iff Q\bar{x} + c = A^T\bar{y} + \bar{s}$ for some $\bar{y}$, $\bar{s} \geq 0$ s.t. $\bar{s}^T\bar{x} = 0$



Recall: $\bar{x} \in C$ $\quad C$ convex

$$N_C(\bar{x}) = \{ g : g^T(y - \bar{x}) \le 0 \quad \forall y \in C \}$$

Opt condns: $\quad \min f(x)$
$\quad\quad\quad\quad\quad\quad x \in C$

$\bar{x}$ solves $\Rightarrow -\nabla f(\bar{x}) \in N_C(\bar{x})$

$\Leftrightarrow \quad \nabla f(\bar{x})^T (y - \bar{x}) \geq 0 \quad \forall y \in C.$

$$\lambda : \mathbb{S}^n \to \mathbb{R}^n$$

$$\mathbb{S}^n_+ \qquad \mathbb{R}^n_+$$

Observe: $X \cdot Y = \text{trace}(XY)$
$$= \sum_{i=1}^{n} \sum_{j=1}^{n} X_{ij} Y_{ij}$$

Property of the trace: $\text{trace}(ABC)$
$$= \text{trace}(BCA)$$

lin inequality eq.

From LP → SDP:

replace $Dx \le d \iff \sum d_j x_j \le d$

with $\sum_{j=1}^{n} F_j x_j \preceq F_0$ ← Linear matrix inequality

Any LP is an SDP:

Observe $x \in \mathbb{R}^n_+ \iff \mathrm{Diag}(x) = \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix} \succeq_c 0$

Observe $c^T x = \mathrm{Diag}(c) \bullet \mathrm{Diag}(x)$

<u>Obs</u>: $\|Y\|_{op} \le 1 \iff YY^T \preceq I_m$

$Y \in \mathbb{R}^{m \times n}$

$\iff \begin{bmatrix} I_m & Y \\ Y^T & I_n \end{bmatrix} \succeq_c 0$

Special case of Schur Complement Thm

$\begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$, A, C symmetric $C \succ 0$

$\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} \succeq_c 0 \iff A - BC^{-1}B^T \succeq_c 0.$

$$\|X\|_{tr} = \max_{Y} \left\{ \operatorname{trace}(X^T Y) : \|Y\|_{op} \leq 1 \right\}$$

$$= \max_{Y} \ \operatorname{trace}(X^T Y)$$

$$\begin{bmatrix} I_m & Y \\ Y^T & I_n \end{bmatrix} \succeq_C 0$$

$$\overset{\text{SDP duality}}{=} \quad \min_{W_1, W_2} \frac{1}{2} \left(I_m \cdot W_1 + I_n \cdot W_2 \right)$$

$$\begin{bmatrix} W_1 & X \\ X^T & W_2 \end{bmatrix} \succeq_C 0$$

$$\left\| \begin{pmatrix} Ax + b \\ c^T x + d \end{pmatrix} \right\| \leq e^T x + f$$

$$\Longleftrightarrow \quad \begin{bmatrix} e^T x + f \\ Ax + b \\ c^T x + d \\ \cancel{e^T x + f} \end{bmatrix} \in Q$$

Can rewrite $\|x\|_2 \le x_0$

in terms of SDP

By Schur Complement Thm

$$\begin{bmatrix} I & x \\ x^T & 1 \end{bmatrix} \succeq 0 \iff \|x\|_2^2 \le 1$$

So $\quad \|x\|_2 \le x_0 \iff \begin{bmatrix} x_0 I & x \\ x^T & x_0 \end{bmatrix} \succeq 0$