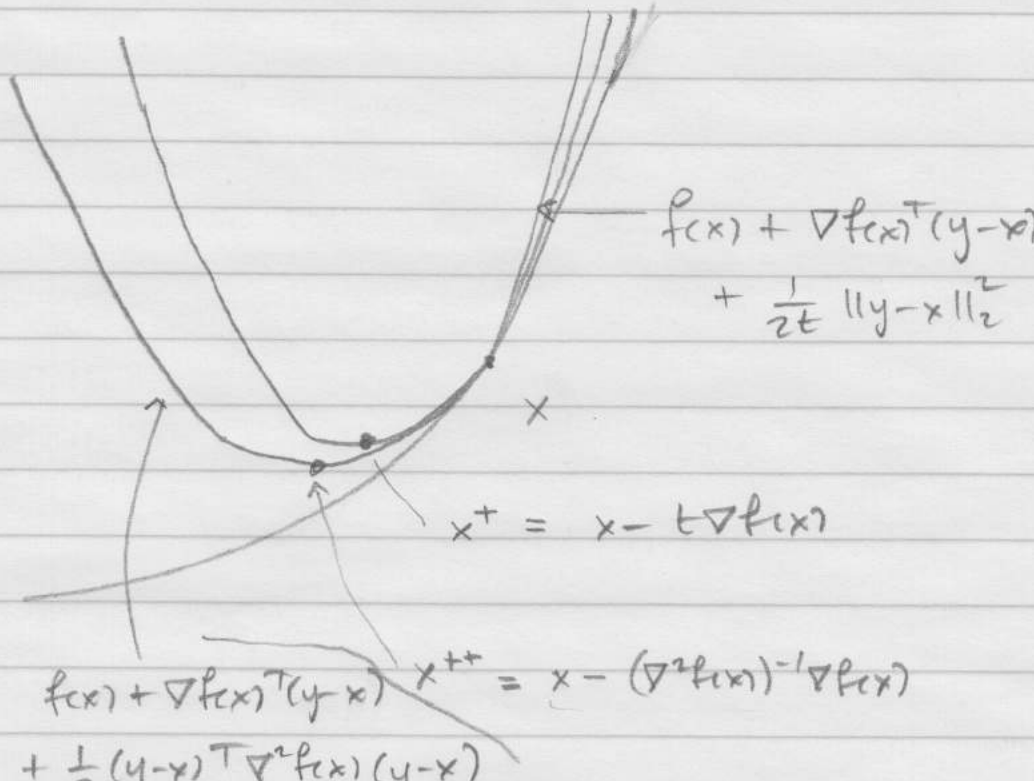
$$\nabla^2 f(x^{(k-1)}) \cdot v = \nabla f(x^{(k-1)})$$

$$H \, v = g$$

$$x^{(k)} = x^{(k-1)} - v$$



Fact: Newton minimizes a quadratic in one step!

$$y^+ = y - A^{-1} (\nabla^2 f(Ay))^{-1} \nabla f(Ay)$$

$$\underbrace{Ay^+}_{x^+} = \underbrace{Ay}_{x} - (\nabla^2 f(\underbrace{Ay}_{x}))^{-1} \nabla f(\underbrace{Ay}_{x})$$

$$\|v\|_A = \sqrt{v^T A v}$$

$$v = -(\nabla^2 f(x))^{-1} \nabla f(x)$$

$$\|v\|_{\nabla^2 f(x)} = \left[\nabla f(x)^T \cancel{(\nabla^2 f(x)^{-1})} \nabla^2 f(x) (\nabla^2 f(x)^{-1}) \nabla f(x) \right]^{1/2}$$

$$= \left[\nabla f(x)^T (\nabla^2 f(x))^{-1} \nabla f(x) \right]^{1/2}$$

$$= \lambda(x)$$

$$\nabla f^2 : \mathbb{R}^n \to \mathbb{R}^{n \times n}$$

$$\| \nabla^2 f(x) - \nabla^2 f(y) \|_F^2 \leq H \|x-y\|_2 .$$

K_0 : number of steps until

$$\| \nabla f(x^{(k+1)}) \|_2 < \eta$$

$$\| \nabla f(x^{(k+1)}) \|_2 \leq \underbrace{\frac{2m^2}{H} \left(\frac{H}{2m^2} \underbrace{\|\nabla f(x^{(k)})\|_2}_{< \eta} \right)^2}$$

$$\leq \frac{H}{2m^2} \eta^2$$

$$\leq \eta/2$$

$$< \eta$$

ε accuracy: $f(x^{(k)}) - f^* \le \varepsilon$

$$\frac{f(x^{(0)}) - f^*}{\gamma}$$

$$f(x) - f^* \le \frac{1}{2m} \|\nabla f(x)\|_2^2$$

$$f(y) \geq f(x) + \nabla f(x)^T (y-x) + \frac{m}{2} \|y-x\|_2^2$$

$$y = x - \nabla f(x)$$

$$\frac{f(x^{(0)}) - f^*}{\gamma} + \underbrace{\log\log\left(\frac{\varepsilon_0}{\varepsilon}\right)}_{\le 5}$$