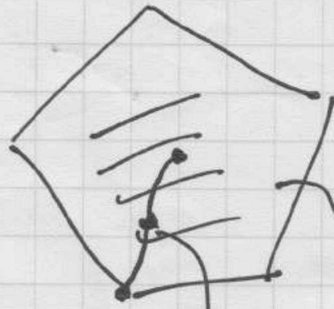


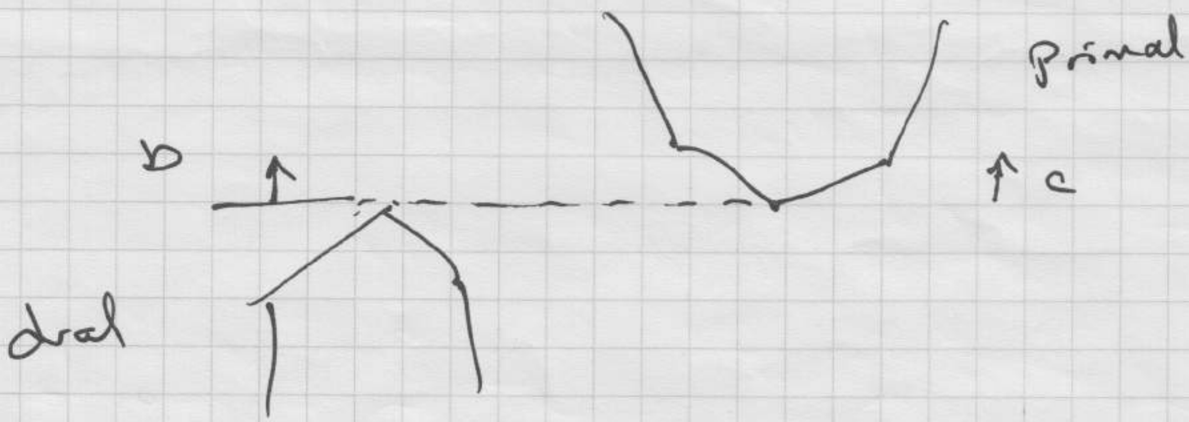
$\{x : h(x) \le 0,\ Ax = b\}$

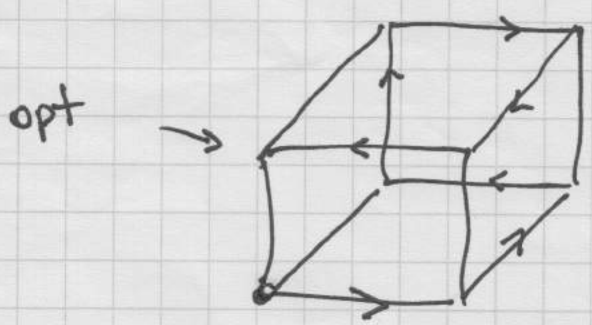
$$x^*(t) = \underset{Ax=b}{\operatorname{argmin}}\ t f(x) + \phi(x)$$

Pf of weak duality thm:

$$c^T x - b^T y = (A^T y + s)^T x - (Ax)^T y$$

$$= s^T x \geq 0.$$





Primal LP

$$\begin{aligned} \min\ & c^T x \\ & Ax = b \\ & x \geq 0 \end{aligned} \quad \rightsquigarrow \quad \begin{aligned} \min\ & c^T x - \tau \sum \log x_j \\ & Ax = b \end{aligned}$$

Barrier problem

Dual LP

$$\begin{aligned} \max\ & b^T y \\ & A^T y + s = c \\ & s \geq 0 \end{aligned} \quad \rightsquigarrow \quad \begin{aligned} \max\ & b^T y + \tau \sum \log s_j \\ & A^T y + s = c \end{aligned}$$

<u>Observe</u>: Take the dual of first problem:

$$L(x,y) = c^T x - \tau \sum \log x_j + y^T (b - Ax)$$

dual: objective: $\max\limits_{x} L(x,y)$ ← finite only if

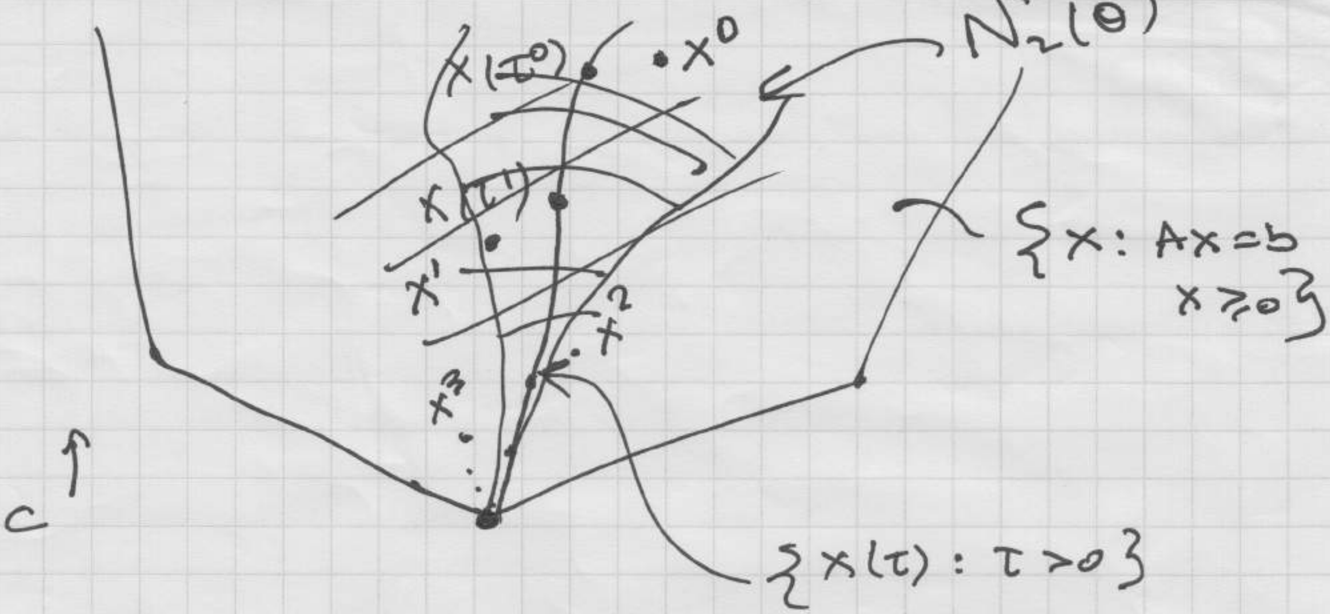
$$L(x,y) = (\underbrace{c - A^T y}_{s > 0})^T x - \tau \sum \log x_j + b^T y$$

$$= \sum s_j x_j - \tau \sum \log x_j + b^T y$$

$$\rightsquigarrow \min_{x} L(x,y) = n\tau - \tau \sum \log \frac{\tau}{s_j} + b^T y$$

$$= n\tau - n\tau \log \tau + b^T y + \tau \sum \log s_j$$

$$\min_{x_j} s_j x_j - \tau \log x_j \leftarrow \text{attained @ } x_j = \frac{\tau}{s_j}$$

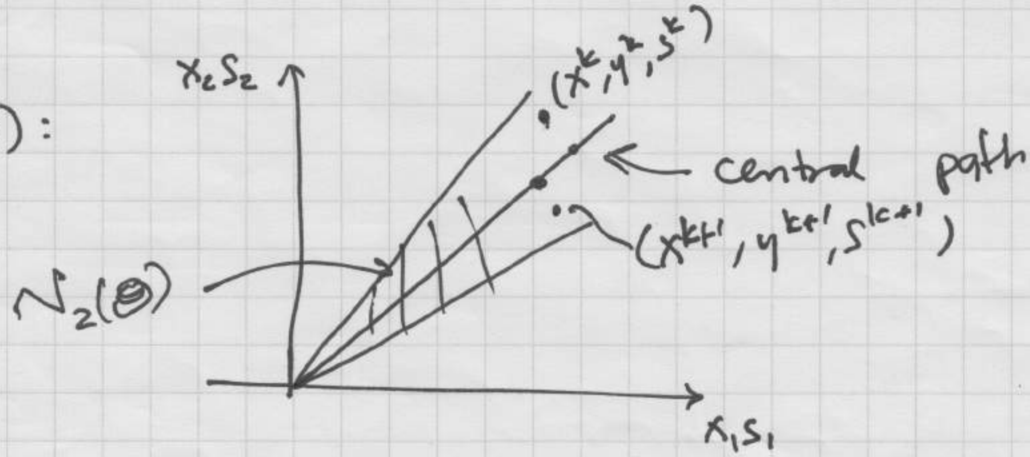


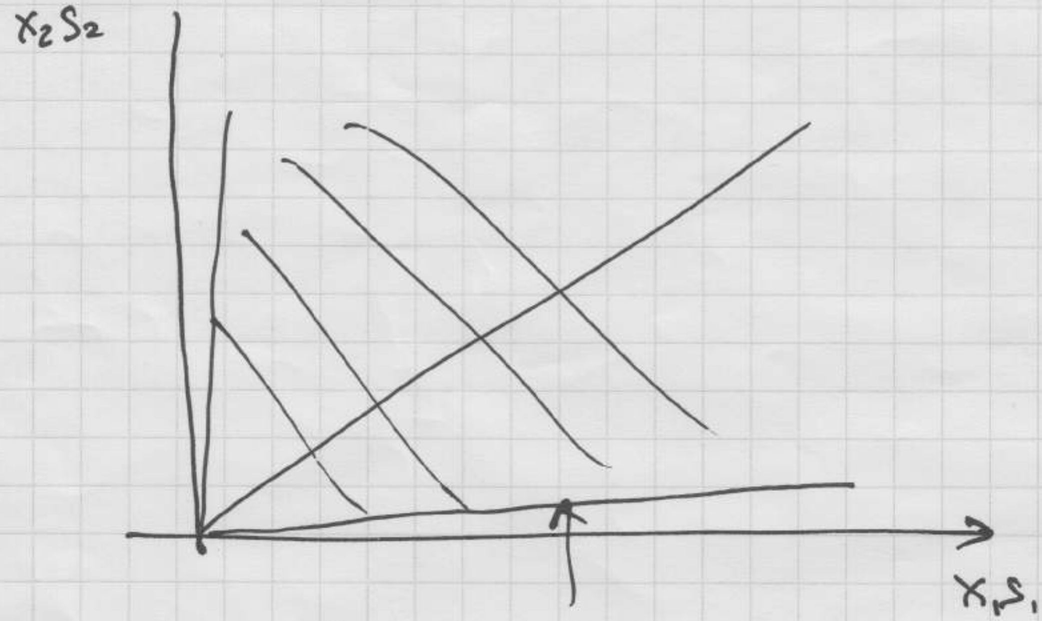
Neat Fact: $\mathcal{F}^0 \qquad \mathbb{R}^n_{++}$

$$(x,y,s) \longmapsto \begin{pmatrix} x_1 s_1 \\ \vdots \\ x_n s_n \end{pmatrix} = XS\mathbb{1}$$

<u>Observe</u>: If $(x,y,s) \in \mathcal{F}^0 \Rightarrow c^T x - b^T y = s^T x = n\mu(x,s)$

$\mathcal{N}_2(\theta)$:





$\mathcal{N}_{-\infty}(\gamma)$

To ensure $\dfrac{\theta^2+\delta^2}{2^{3/2}(1-\theta)} \le \left(1-\dfrac{\delta}{\sqrt{n}}\right)\theta$

can choose eg $\theta = \delta = 0.4$

$x^+ = x^{k+1}$, $y^+ = y^{k+1}$, $s = s^{k+1}$, $x^k = x$, etc.

$$x^+ = x + \Delta x \qquad y^+ = y + \Delta y \qquad s^+ = s + \Delta s$$

$$Ax^+ = Ax + A\Delta x = b + 0 = b \checkmark$$

likewise $A^T y^+ + s^+ = A^T y + s + A^T \Delta y + \Delta s = c + 0 = c \checkmark$

$$\begin{aligned}
X^+ S^+ \mathbb{1} &= (X + \Delta X)(S + \Delta S)\mathbb{1} \\
&= \underbrace{X \Delta s + S \Delta x} + \Delta X \Delta S \mathbb{1} + XS\mathbb{1} \\
&= \tau \mathbb{1} - \cancel{XS\mathbb{1}} + \Delta X \Delta S \mathbb{1} + \cancel{XS\mathbb{1}} \\
&= \tau \mathbb{1} + \Delta X \Delta S \mathbb{1}
\end{aligned}$$

$$\begin{aligned}
\mu(x^+, s^+) &= \frac{1}{n}(x^+)^T(s^+) = \frac{1}{n}\left[n\tau + \Delta x^T \Delta s\right] \\
&= \tau + \frac{1}{n}\Delta x^T \Delta s \\
&= \underline{\left(1 - \frac{\delta}{\sqrt{n}}\right)\mu(x,s)} + \frac{1}{n}\cancelto{0}{\Delta x^T \Delta s}
\end{aligned}$$

Look at Newton Step eqns:

$$\left.\begin{aligned} A^T \Delta y + \Delta s &= 0 \\ A\Delta x &= 0 \end{aligned}\right\} \Rightarrow \cancelto{0}{\Delta x^T A^T \Delta y} + \Delta x^T \Delta s = 0$$

After k itns:

$$\mu(x^k, s^k) = \left(1 - \frac{\delta}{\sqrt{n}}\right)^k \mu(x^0, s^0)$$

$\rightsquigarrow$ in $O\left(\sqrt{n} \log \frac{n\mu(x^0, s^0)}{\varepsilon}\right)$

get x^k, (y^k, s^k) st

$c^T x^k - b^T y^k = n\mu(x^k, s^k) < \varepsilon.$