$$\min_{x \in C} \quad \underset{\substack{\uparrow \\ \text{smooth}}}{g(x)} + \underset{\substack{\uparrow \\ \text{nonsmooth}}}{h(x)}$$

$$\Longleftrightarrow \min_{x} \quad g(x) + \underbrace{h(x) + 1_C(x)}_{\tilde{h}(x)}$$

$$\text{prox}_{\tilde{h},t}(x) = \underset{z}{\text{argmin}} \ \frac{1}{2t} \|x - z\|_2^2 + \tilde{h}(z)$$

$$= \underset{z \in C}{\text{argmin}} \ \frac{1}{2t} \|x - z\|_2^2 + h(z)$$

$$x^+ = \text{prox}_{\tilde{h},t}(x - t \nabla g(x))$$

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6. Derive dual of

$$\min_{x} \quad \sum_{i=1}^{N} \|\underbrace{A_i x}_{z_i} + b_i\|_1 + \frac{1}{2}\|x\|_2^2$$

$$\Longleftrightarrow \min_{x, z_i} \quad \sum \|z_i + b_i\|_1 + \frac{1}{2}\|x\|_2^2$$

$$\text{s.t.} \quad A_i x = z_i, \text{ all } i$$

$$L(x, z_i, u_i) = \sum \|z_i + b_i\|_1 + \frac{1}{2}\|x\|_2^2 + \sum u_i^T (z_i - A_i x)$$

$$= \underbrace{\sum_{i=1}^{N} \left(\|z_i + b_i\|_1 + u_i^T z_i \right)}_{\min_{z_i}(\ )} + \underbrace{\frac{1}{2}\|x\|_2^2 - \left(\sum A_i^T u_i \right)^T x}_{\min_x(\ )}$$

$\min_x (\ ) = -\frac{1}{2} \| \sum A_i^T u_i \|_2^2$

$\min_{z_i} (\ ) = \underbrace{\min_{y_i} \|y_i\|_1 + u_i^T y_i - u_i^T b_i}$

$y_i = z_i + b_i$

$= -\mathbb{1}\{ \| -u_i \|_\infty \le 1 \}$

$= -\mathbb{1}\{ \| u_i \|_\infty \le 1 \}$

dual problem

$$\max_{u_i} \ -\sum u_i^T b_i - \frac{1}{2} \| \sum A_i^T u \|_2^2$$

$$\text{st} \quad \|u_i\|_\infty \le 1 \ \text{all } i$$

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1. $\min_{y_1, y_2} \quad -4y_1 + 2y_2$ ← Dual of and specify duality gap

st. $-y_1 + y_2 \ge 2$

$y_1 - y_2 \ge 1$

$y_1, y_2 \ge 0$

Primal infeasible

$y_1 - y_2 \le -2$

$-y_1 + y_2 \le -1$

$-y_1, -y_2 \le 0$

$L(y_1, y_2, u_1 \ldots u_4) = -4y_1 + 2y_2 + u_1(y_1 - y_2 + 2) + u_2(-y_1 + y_2 + 1)$
$- u_3 y_1 - u_4 y_2$

$= y_1 \underbrace{(-4 + u_1 - u_2 - u_3)} + y_2 \underbrace{(2 - u_1 + u_2 - u_4)} + 2u_1 + u_2$

dual: $\max\limits_{u_1 .. u_4} \quad 2u_1 + u_2$

s.t. $-4 + u_1 - u_2 - u_3 = 0$

$2 - u_1 + u_2 - u_4 = 0$

$u_1 .. u_4 \geq 0$

$-2 - u_3 - u_4 = 0$

$u_3 + u_4 = -2$

dual infeasible

duality gap $= \infty$.