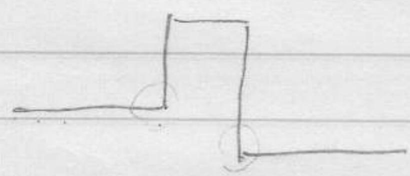
$$D = I = \begin{bmatrix} - e_1 - \\ \vdots \\ - e_n - \end{bmatrix}$$

$$D = \begin{bmatrix} - d_1 - \\ \vdots \\ - d_m - \end{bmatrix}$$

$$D\beta = \begin{bmatrix} \beta_2 - \beta_1 \\ \beta_3 - \beta_2 \\ \vdots \\ \beta_n - \beta_{n-1} \end{bmatrix}$$ Fused lasso, 1d.

many $\beta_i = \beta_{i+1}$ if $D\beta$ is sparse



$$\min \ \underbrace{\tfrac{1}{2} \Sigma (y_i - \beta_i)^2}_{f(\beta)} + \lambda \underbrace{\Sigma |\beta_i - \beta_{i+1}|}_{\|D\beta\|_1}$$ Fused lasso, 1d

$$p_i = \frac{1}{1 + e^{-\beta_i}}$$

$$\hat{\beta}_i = \hat{f}\left(\frac{i}{n}\right)$$

$$\|D\beta\|_1 = \sum |\beta_i - 2\beta_{i+1} + \beta_{i+2}|$$

many $\beta_{i+1} = \frac{\beta_i + \beta_{i+2}}{2}$ if $D\beta$ is sparse

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$$D = \begin{bmatrix} D^{const} \\ D^{quad} \end{bmatrix}$$

$$\mu_i = e^{\beta_i}$$

$$\min_{\beta} \sum_{i \text{ obs}} (y_i - \beta_i)^2 + \lambda \sum_{\substack{(i,j) \\ \in E}} |\beta_i - \beta_j| + \varepsilon \|\beta\|_2^2$$

$$\partial g(x) = A^T \partial f(Ax)$$

$$g(x) = f(Ax+b)$$

$$\gamma_i \in \begin{cases} \{\text{sign}((D\beta)_i)\} & \text{if } (D\beta)_i \neq 0 \\ [-1, 1] & \text{if } (D\beta)_i = 0 \end{cases}$$

$$D\beta = \alpha^+ - \alpha^-$$

$$\min_{\beta,\alpha} \quad f(\beta) + \lambda \mathbb{1}^T(\alpha^+ + \alpha^-)$$

$$\text{st.} \quad \left.\begin{aligned} D\beta &= \alpha^+ - \alpha^- \\ \alpha^+, \alpha^- &\geq 0. \end{aligned}\right\}$$

$$\min_{\beta,z} \quad f(\beta) + \lambda\|z\|_1$$

$$\text{st} \quad D\beta = z$$

$$L(\beta,z,u) = f(\beta) + \lambda\|z\|_1 + u^T(D\beta - z)$$

$$\min_{\beta,z} L(\beta,z,u) = -f^*(-D^Tu) - I\left(\|u\|_\infty \leq \lambda\right)$$

$$\nabla f(\hat{\beta}) + D^T\hat{u} = 0$$

$$\lambda\gamma - u = 0, \quad \gamma \in \partial\|z\|_1$$

$$\Longleftrightarrow \quad u \in \lambda \cdot \partial\|x\|_1 \Big|_{x = D\hat{\beta}}$$