$$s_j = b_j - \left(\Sigma A_i x_i\right)_j$$

$$\beta^+ = \underset{\beta}{\operatorname{argmin}} \ \tfrac{1}{2}\|y - X\beta\|_2^2 + \tfrac{\rho}{2}\|\beta - \alpha + \omega\|_2^2$$

$$= (X^T X + \rho I)^{-1}(X^T y + \rho(\alpha - \omega))$$

$$\alpha^+ = \underset{\alpha}{\operatorname{argmin}} \ \Sigma \|\alpha_{I_g}\|_2 + \tfrac{\rho}{2}\|\beta - \alpha + \omega\|_2^2$$

$$\Longleftrightarrow \quad \alpha_{I_g} = R_{\lambda/\rho}\left(\beta_{I_g} + \omega_{I_g}\right)$$

$$R_t(x) = \left(1 - \frac{t}{\|x\|}\right)_+ x$$

$$\omega^+ = \omega + \beta - \alpha$$

$$\max_{u} \underbrace{-f^*(-A^T u)}_{g(u)} - b^T u$$

sub.gs: $\partial g(u) - b.$

$$\partial g(u) = A \, \underbrace{\partial f^*(-A^T u)}$$

$$x \in \partial f^*(-A^T u) \iff x \in \operatorname*{argmin}_{z} f(z) + u^T A z$$

$u^{(0)}$

$$\begin{cases} \boxed{x^{(k)} \in \operatorname*{argmin}_{z} f(z) + u^{(k-1)T} A z} \\ u^{(k)} = u^{(k-1)} + t_k (A x^{(k)} - b) \end{cases}$$

$$x^* \in \operatorname*{argmin}_{x} L(x, u^*)$$

$\Rightarrow$ x^* primal solution

$f(x)$ st. convex with param d

$f(x) - u^T x$ is too

$f(x) - v^T x$ is too

∇f^* Lipschitz with param $1/d$

$\Rightarrow$ g.d. with step size $1/\text{Lip} = d$
converges at rate $O(1/\varepsilon)$

---

f strong convex and ∇f Lipschitz

$\Rightarrow$ g.d. on f gets rate $O(\log 1/\varepsilon)$

still.

$\Rightarrow$ g.a on dual (involves f^*) gets
rate $O(\log(1/\varepsilon))$

$$\min_x \sum_{i=1}^{B} f_i(x_i) \quad \text{s.t.} \quad \sum_{i=1}^{B} A_i x_i = b$$

$$x^+ \in \underset{\substack{z \\ =(z_1 .. z_B)}}{\operatorname{argmin}} \sum f_i(z_i) + u^T\left(\sum A_i z_i\right)$$

$$x_i^+ \in \underset{z_i}{\operatorname{argmin}}\ f_i(z_i) + u^T A_i z_i$$

SEPARABLE over $i = 1 .. B$