Quiz 1

Convex Optimization, 10-725

Due Friday September 13, 2019

Name:

Andrew ID:

Each question is either in true/false format, or multiple choice. For multiple choice, just choose the single best option. In each case, make sure to fill in the box according to the answer you choose (true or false, or the multiple choice option) completely. All questions are worth 1 point.

1. Strong convexity implies strict convexity.
   ☐ True
   ☐ False

2. If f is convex, then f has convex sublevel sets, $\{x : f(x) \leq t\}$, for all $t \in \mathbb{R}$.
   ☐ True
   ☐ False

3. Strong convexity implies differentiability.
   ☐ True
   ☐ False

4. If f is a strictly convex function, then it must attain its minimum and its minimizer must be unique.
   ☐ True
   ☐ False

5. For a twice differentiable function f, the function f is strictly convex if and only if $\nabla^2 f(x) \succ 0$ for all x.
   ☐ True
   ☐ False

6. The convex hull of a closed set is always closed (and convex).
   ☐ True
   ☐ False

7. A function f is convex if and only if its epigraph $\{(x, t) : f(x) \leq t\}$ is a convex set.
   ☐ True
   ☐ False

8. The function $f(x) = \|y - Ax\|_2^2$ is convex (even if A is square and has negative eigenvalues).
   ☐ True
   ☐ False

9. For a closed, nonconvex set C, the projection of any point x onto C is always unique.
   ☐ True
   ☐ False

10. The convex hull of finitely many points is always convex, but the convex hull of infinitely many need not be.
    ☐ True
    ☐ False

11. A set $C \subseteq \mathbb{R}^n$ is convex if for every pair of points, $x, y \in C$, the line segment between them is also contained in C. Formally, this is:
    ☐ a. $tx + (1-t)y \in C$, for all $t \in \mathbb{R}$;
    ☐ b. $tx + (1-t)y \in C$, for all $t \in [0, 1]$;

☐ c. $tx + ty \in C$, for all $t \in [0, 1]$;
☐ d. $tx + ty \in C$, for all $t \in \mathbb{R}$.

12. The function g defined by partially minimizing f, i.e., $g(x) = \min_{y \in C} f(x, y)$, is convex whenever f is convex.
☐ True
☐ False

13. A point x minimizes a convex, differentiable function f over a convex set C if and only if:
☐ a. $\nabla f(x)^T (y - x) \geq 0$ for all $y \in C$;
☐ b. $\nabla f(x)^T (x - y) \geq 0$ for all $y \in C$;
☐ c. $f(y) \geq f(x) + \nabla f(y)^T (y - x)$ for all $y \in C$;
☐ d. $(\nabla f(y) - \nabla f(x))^T (x - y) \geq 0$ for all $y \in C$.

14. A convex optimization problem must have zero, one, or infinitely many minimizers (no other number is possible).
☐ True
☐ False

15. In a convex optimization problem, any local minimizer is automatically globally optimal.
☐ True
☐ False

16. A convex optimization problem cannot have more than one minimizer.
☐ True
☐ False

17. For a point x to be considered feasible with respect to a given optimization problem, which of the following need not be true about x?
☐ a. $x \in D$, where D is the intersection of domains of functions defining the optimization problem;
☐ b. x minimizes the objective function;
☐ c. x satisfies the constraints of the objective function;
☐ d. none of the above.

18. For an LP, the two problem forms:

$$\begin{aligned} \min_{y} \quad & d^T y \\ \text{subject to} \quad & Gy \leq e, \; Hy = f \end{aligned}$$

and

$$\begin{aligned} \min_{x} \quad & c^T x \\ \text{subject to} \quad & Ax = b, \; x \geq 0 \end{aligned}$$

are equivalent, meaning that any problem that can be represented in one form can be represented in the other.
☐ True
☐ False

19. A linear program is always a convex optimization problem.
☐ True
☐ False

20. For an SDP, the two problem forms

$$\begin{aligned} \min_{x} \quad & c^T x \\ \text{subject to} \quad & x_1 F_1 + \ldots + x_n F_n \preceq F_0 \\ & Ax = b \end{aligned}$$

and

$$\begin{aligned} \min_{X} \quad & C \bullet X \\ \text{subject to} \quad & A_i \bullet X = b_i, \; i = 1, \ldots, m \\ & X \succeq 0 \end{aligned}$$

are equivalent, meaning that any problem that can be represented in one form can be represented in the other.
☐ True
☐ False