Quiz 3

Convex Optimization, 10-725

Due Friday October 11, 2019

Name:

Andrew ID:

Each question is either in true/false format, or multiple choice. For multiple choice, just choose the single best option. In each case, make sure to fill in the box according to the answer you choose (true or false, or the multiple choice option) completely. All questions are worth 1 point.

1. The dual of a linear program is another linear program.
   ☐ True
   ☐ False

2. A linear program can have duality gap equal to 1.
   ☐ True
   ☐ False

3. The duality gap in a linear program is always zero.
   ☐ True
   ☐ False

4. The dual of a quadratic program is another quadratic program.
   ☐ True
   ☐ False

5. A convex optimization problem always has a dual problem that can be expressed in closed-form.
   ☐ True
   ☐ False

6. Strong duality always holds in a convex optimization problem.
   ☐ True
   ☐ False

7. The dual problem is always easier to solve than the primal, but it is simply not always calculable.
   ☐ True
   ☐ False

8. The dual is always a convex optimization problem, no matter the original primal problem.
   ☐ True
   ☐ False

9. Strong duality, in convex optimization:
   ☐ a. holds for LPs, but not in generality, beyond this class;
   ☐ b. holds for LPs and QPs, but not in generality, beyond these classes;
   ☐ c. holds when there exists a strictly feasible point (satisfies all inequality constraints strictly, and satisfies equality constraints);
   ☐ d. always holds.

10. The dual problem:
    ☐ a. always has more variables than the primal;
    ☐ b. always has less variables than the primal;
    ☐ c. is always easier to solve than the primal, but it is simply not always calculable;
    ☐ d. none of the above.

11. Expressing a primal solution in terms of a dual solution can be done with:
    ☐ a. the KKT primal and dual feasibility conditions;
    ☐ b. the KKT complementary slackness condition, under strong duality;
    ☐ c. the KKT stationarity condition, always;
    ☐ d. the KKT stationarity condition, under strong duality.

12. The KKT conditions are sufficient for optimality, even for a nonconvex optimization problem.
    ☐ True
    ☐ False

13. The KKT conditions give a direct solution to the problem of minimizing a quadratic function subject to equality constraints.
    ☐ True
    ☐ False

14. If f is strictly convex, then $\nabla f^*(y) = \mathrm{argmin}_x \, (f(x) - y^T x)$.
    ☐ True
    ☐ False

15. The dual of $\min_x \; f(x) + g(x)$, in terms of the conjugates f^*, g^* of f, g, respectively, is:
    ☐ a. $\max_u \; f^*(u) + g^*(-u)$;
    ☐ b. $\max_u \; f^*(u) + g^*(u)$;
    ☐ c. $\max_u \; -f^*(u) - g^*(u)$;
    ☐ d. $\max_u \; -f^*(u) - g^*(-u)$.

16. The proximal operator for any norm $\|\cdot\|$ can always be written in closed form, in terms of its dual norm $\|\cdot\|_*$.
    ☐ True
    ☐ False

17. The conjugate f^* of $f(x) = \frac{1}{2}x^T Q x$, where $Q \succ 0$, is defined by:
    ☐ a. $f^*(y) = -\frac{1}{2}y^T Q^{-1} y$;
    ☐ b. $f^*(y) = \frac{1}{2}y^T Q^{-1} y$;
    ☐ c. $f^*(y) = \frac{1}{2}y^T Q y$;
    ☐ d. none of the above.

18. If f is convex and closed, then $y \in \partial f(x) \iff x \in \partial f^*(y)$.
    ☐ True
    ☐ False

19. For a problem of the form $\min_x \; f(x) + g(Ax)$, duality can "shift" the appearance of the linear transformation A, meaning that A will appear in the conjugate of f in the dual criterion, rather than in the conjugate of g.
    ☐ True
    ☐ False

20. The conjugate f^* of a function f is always convex.
    ☐ True
    ☐ False